Mathematical Models

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Visuality Workshop
December 6, 2010

Summer Project

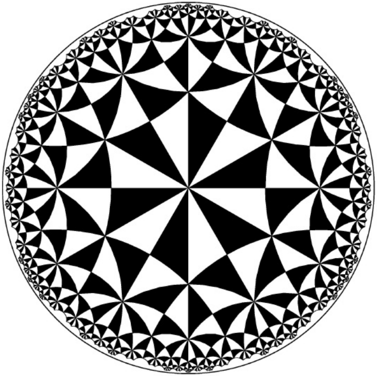
- Hired two studio art students to produce "mathematical art" useful in topology and geometry.
- Chose from various projects, according to interests and strengths.
- Jacque Oman '11 crocheted and quilted, following set of instructions found via internet and from colleague (Jeff Weeks).
- Krissy Lunz '12 made sculptures in a variety of media, using ideas from mathematical texts.

Topology and Geometry: A brief overview

In topology and geometry, we often study 2-dimensional surfaces like the plane, sphere, and torus. There are many other types of surfaces, but some of these cannot be easily visualized using paper drawings. Mathematically, we are interested in the properties of these surfaces as well as using them as tools to help analyse other objects, like knots and 3-dimensional spaces.

Hyperbolic planes

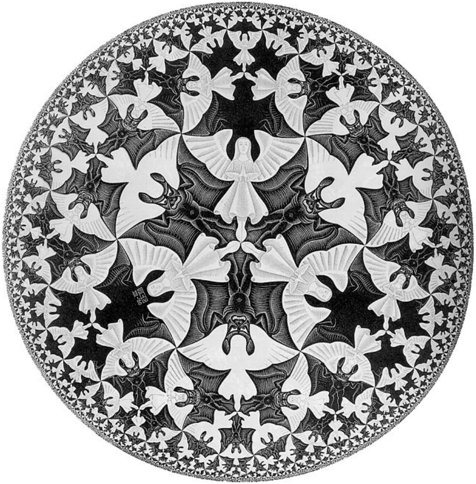
The hyperbolic plane is a model of a geometric system where Euclid's Parallel Postulate is not assumed.



One model of the hyperbolic plane is using a disk. Here is Coxeter's tesellation, using hyperbolic triangles (of equal size and area).

Hyperbolic planes

Escher's Circle Limit V was based on Coxeter's mathematics.



Hyperbolic planes (Jacque Oman)

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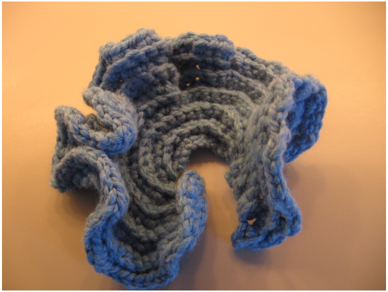
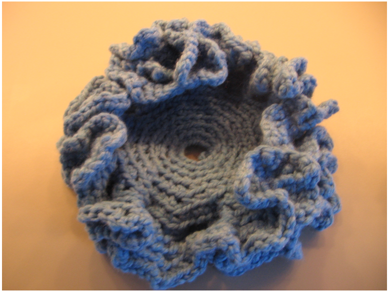




Hyperbolic planes

Jacque created these objects following instructions obtained from an article in the Mathematical Intelligencer by D. Henderson and D. Taimida.

In order to make the crocheted hyperbolic plane you need just a very basic crocheting skills. All you need to know is how to make a chain (to start) and how to single crochet. That's it! Now you can start. See Figure 2 for a picture of these stitches, which will be described further in the next paragraph.

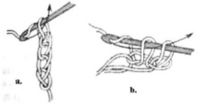
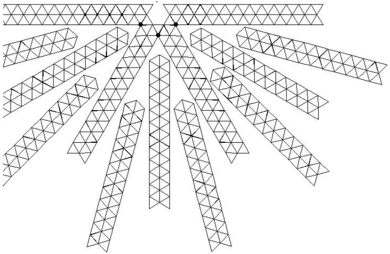


Figure 2. Crochet stitches for the hyperbolic plane.

First you should chose a yarn which will not stretch a lot. Every yarn will stretch a little but you need one which will keep its shape. Now you are ready to start the stitches:

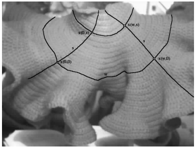
1. Make your **beginning chain stitches** (Figure 2a). (Topologists may recognize that as the stitches in the Fox-Artin wild arc!) About 20 chain stitches for the beginning will be enough.
2. **For the first stitch in each row** insert the hook into the 2nd chain from the hook. Take yarn over and pull through chain, leaving 2 loops on hook. Take yarn over and pull through both loops. One single crochet stitch has been completed. (Figure 2b.)
3. **For the next N stitches** proceed exactly like the first stitch except insert the hook into the next chain (instead of the 2nd).
4. **For the (N+1)st stitch** proceed as before except insert the hook into the same loop as the N-th stitch.
5. **Repeat Steps 3 and 4** until you reach the end of the row.
6. **At the end of the row** before going to the next row do one extra chain stitch.
7. **When you have the model as big as you want,** you can stop by just pulling the yarn through the last loop.

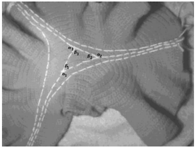


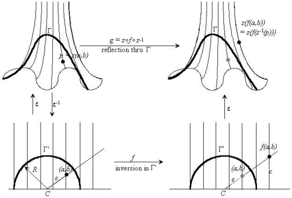
D. Henderson, D. Taimida, "Crocheting the Hyperbolic Plane", Math. Intell., Vol. 23, No. 2, 17-28, 2001.

Hyperbolic planes

These models of the hyperbolic plane will hopefully be useful in an upcoming Senior Seminar on Hyperbolic Surfaces in Spring 2011.



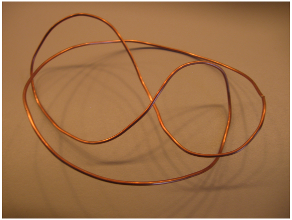


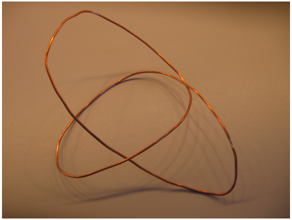


D. Henderson, D. Taimida, "Crocheting the Hyperbolic Plane", Math. Intell., Vol. 23, No. 2, 17-28, 2001.

Knots (Krissy Lunz)

Knot theory is another area of topology. It has applications to studies of 3-dimensional spaces as well as to many of the sciences, e.g., quantum physics and circular DNA biology.





Seifert surface of a knot (Krissy Lunz)

The boundary of a Seifert surface is a knot. There are many possible Seifert surfaces for a given knot, of different topological complexity and with various properties.

Seifert surface of a knot

Seifert developed an algorithm for creating Seifert surfaces, but textbook descriptions are often hard to follow. Pictorial descriptions can also often be confusing.

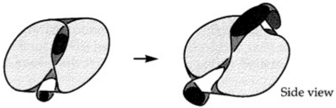


Figure 4.55 Connect the disks by twisted bands.

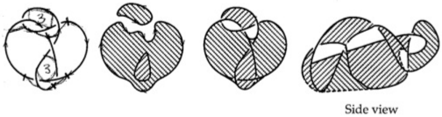


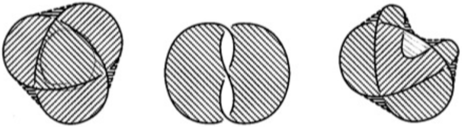
Figure 4.56 This surface has boundary the 6_3 knot.

C. Adams, The Knot Book, AMS, 2004.

Seifert surface of a knot

Drawings of (orientable and non-orientable) Seifert surfaces of a trefoil knot used to produce Krissy's metal sculptures.

A Möbius band with boundary the trefoil knot.



4.59 Other Seifert surfaces for the same knot.

C. Adams, The Knot Book, AMS, 2004.

Chalkboard surfaces (Krissy Lunz)

These chalkboard surfaces allow us to make illustrations that are impossible on a planar chalkboard. They were particularly requested by my comps students from 2009-10 (A. Fisher, R. Phelps, D. Wells) who studied operations on knots drawn on the torus surface.

Reflections

- Difficult to predict the amount of time needed to produce each object.
- Easier with explicit instructions, e.g. obtained from articles, internet images, or textbooks.
- Need studio art student also with mathematical expertise for more free-form projects.

Thanks to

- Margaret Pezalla-Granlund
- Fred Hagstrom
- Ross Elfline