PHYSICS 1202.200 Group Problem 5

This is a closed book, closed notes problem. Calculators are permitted. The **ONLY** formulas that may be used are those given below. Define all symbols and justify all mathematical expressions used. Make sure to state all of the assumptions used to solve a problem.

Useful Mathematical Relationships:

If $Ax^2 + Bx + C = 0$, then $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$, for small angles $\sin\theta = \tan\theta = \theta$

$\frac{d(z^n)}{dz} = nz^{n-1}$, $\frac{d(\cos z)}{dz} = -\sin z$, $\frac{d(\sin z)}{dz} = \cos z$, $\frac{d(e^{az})}{dz} = ae^{az}$, $\frac{d(\ln z)}{dz} = \frac{1}{z}$, $\frac{df(z)}{dt} = \frac{df(z)}{dz}\frac{dz}{dt}$,

Fundamental Concepts, Principles, and Definitions:

$\sum \vec{F} = m\vec{a}$	$\rho = \frac{m}{V}$	$E_f - E_i = E_{in} - E_{out}$	$KE = \frac{1}{2}mv^2$	$P = \frac{F}{A}$	$e = \frac{E_{desired}}{E_{input}}$
$\tau = rF_\perp$	$\frac{dW}{d\ell} = F_\ell$	$f = \frac{1}{T}$	$\frac{dU}{dx} = -F_{internal}$	$S = k\ln\Omega$	$\theta = \frac{\delta C}{r}$
$F = U - TS$	$\frac{dx}{dt} = v_x$	$\frac{dv_x}{dt} = a_x$	$s_{av} = \frac{distance}{\Delta t}$	$v_{x\,av} = \frac{\Delta x}{\Delta t}$	$a_{x\,av} = \frac{\Delta v_x}{\Delta t}$
$\frac{dq}{dt} = I$	$E = qV$	$\frac{dE}{dt} = P$	$q_f - q_i = q_{in} - q_{out}$	$q = C\Delta V$	

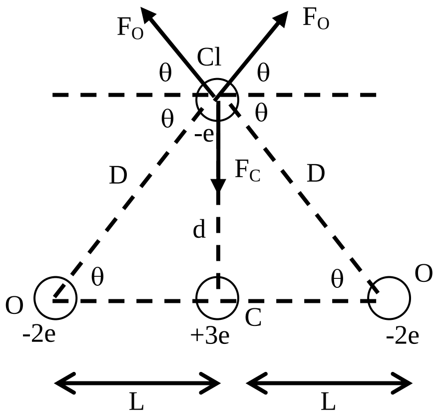
Under Certain Conditions:

$F = mg$	$F = kx$	$F = \mu_k n$	$F \le \mu_s n$	$F = G\frac{m_1 m_2}{r^2}$	$F = k_e\frac{q_1 q_2}{r^2}$
$\sum \tau = 0$	$\Delta E_{internal} = mL$	$\Delta E_{internal} = Cm\Delta T$	$PV = NkT$	$PV = nRT$	$\frac{dW}{dV} = P$
$U = mgh$	$U = \frac{1}{2}kx^2$	$W = -T\Delta S$	$\Delta S = \frac{Q}{T}$	$F = bv$	$a = \frac{v^2}{r}$
$x = A\cos(2\pi ft + \phi)$	$x = \frac{1}{2}at^2 + v_o t + x_o$	$\frac{1}{2}\rho v^2 + \rho gy + P = constant$		$n_1 \sin\theta_1 = n_2 \sin\theta_2$	
$\Delta V = IR$	$P = I\Delta V$	$E = \frac{1}{2}C\Delta V^2$			

Useful constants: $k = 9.0 \times 10^9\ Nm^2/C^2$

You are working in a lab doing research on atmospheric pollution. Your group is trying to determine what unusual types of chlorinated molecules might exist in the upper atmosphere. You have been asked to determine where a chlorine ion of effective charge -e would situate itself near a carbon dioxide ion. The carbon dioxide ion you are investigating is composed of 2 oxygen ions, each with an effective charge -2e, and a carbon ion with an effective charge +3e. These ions are arranged in a line with the carbon ion sandwiched midway between the two oxygen ions. The distance between each oxygen ion and the carbon ion is 3.0×10^{-11} m. Assuming that the chlorine ion is on a line perpendicular to the axis of the carbon dioxide ion and that the line goes through the carbon ion, what is the equilibrium distance for the chlorine ion relative to the carbon ion on this line? For simplicity, you assume that the carbon dioxide ion does not deform in the presence of the chlorine ion. Looking in your trusty physics textbook, you find the charge of the electron is 1.60×10^{-19} C.

Solution – Group Problem 5



$L = 3.0 \times 10^{-11}$ m
$e = 1.60 \times 10^{-19}$ C

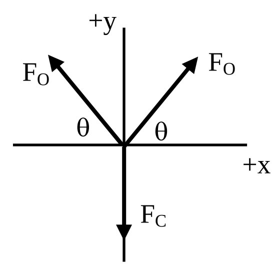
Question: What is position of the chlorine ion from the carbon ion on a line perpendicular to the axis of the carbon dioxide ion when the sum of the forces on the chlorine ion is zero?

Approach:
Find the sum of the forces on the Cl ion.
Need to get components. Choose a coordinate system with one axis along the CO_2 axis.
Need to define some angles in the picture.

Forces are electric. $F = k_e \frac{q_1 q_2}{r^2}$

$$F_O = k_e \frac{(2e)(e)}{D^2} = 2k_e \frac{e^2}{D^2}$$

$$F_C = k_e \frac{(3e)(e)}{d^2} = 3k_e \frac{e^2}{d^2}$$



$$\sum F_x = 2k_e \frac{e^2}{D^2}\cos\theta - 2k_e \frac{e^2}{D^2}\cos\theta = 0$$

$$\sum F_y = 2k_e \frac{e^2}{D^2}\sin\theta + 2k_e \frac{e^2}{D^2}\sin\theta - 3k_e \frac{e^2}{d^2} = 0$$

From the picture: $\sin\theta = \frac{d}{D}$

$$2k_e \frac{e^2}{D^2}\frac{d}{D} + 2k_e \frac{e^2}{D^2}\frac{d}{D} - 3k_e \frac{e^2}{d^2} = 0$$

$$4\frac{1}{D^3}d - 3\frac{1}{d^2} = 0$$

$$4\frac{1}{D^3}d^3 = 3$$

$$\frac{d}{D} = \left(\frac{3}{4}\right)^{\frac{1}{3}}$$

from the picture:

$$d^2 + L^2 = D^2$$

$$\left(d^2 + L^2\right)^{\frac{1}{2}} = D$$

$$\frac{d}{\left(d^2 + L^2\right)^{\frac{1}{2}}} = \left(\frac{3}{4}\right)^{\frac{1}{3}}$$

$$d = \left(\frac{3}{4}\right)^{\frac{1}{3}}\left(d^2 + L^2\right)^{\frac{1}{2}}$$

$$d^2 = \left(\frac{3}{4}\right)^{\frac{2}{3}}\left(d^2 + L^2\right)$$

$$d^2 - \left(\frac{3}{4}\right)^{\frac{2}{3}}d^2 = \left(\frac{3}{4}\right)^{\frac{2}{3}}L^2$$

$$(1 - 0.83)d^2 = 0.83L^2$$

$$d^2 = 4.9L^2$$

$$d = 2.2L$$

$$d = 2.2\left(3.0 \text{ x } 10^{-11} \text{ m}\right)$$

$d = 6.6 \text{ x } 10^{-11}$ m

The units are correct since distance is in meters.

The distance is not unreasonable since it is of the same order of magnitude as the other distance in the problem, the length of the CO_2 molecule.