Notes for Solow (1957)

Solow, Robert. 1957. "Technical change and the aggregate production function." ***Review of Economics and Statistics,*** **Vol. 39:3, pp. 312-320.**

Theory:

Solow's (1957) aggregate production function model:

> Q= real GDP
> A= 'neutral technical change'
> K = real capital stock
> L = real employment

Solow estimates a Cobb-Douglas production function for the US. There are two fundamental assumptions: (1) inputs are paid their marginal product (MPL = wage rate, etc…this assumption comes straight from the profit maximization condition for perfectly competitive firms. *Why?*); (2) there are constant returns to scale in the production function:

(Solow's eqn. 1a) $Q_t = A_t K_t^{\alpha} L_t^{\beta}$, where $\boldsymbol{\alpha + \beta = 1}$.

Since the "factor shares" (α, β) add up to 1, we have constant returns to scale *(why?)*.

Since this is non-linear, we use logs to get :

$$\ln Q_t = \ln A_t + \alpha \ln K_t + \beta \ln L_t .$$

> Note: if we wanted to know how output changes over time, all we have to do is to take the total derivative of this equation:
>
> $$d \ln Q_t = \frac{1}{Q} dQ = \frac{dQ}{Q} = \frac{\dot{Q}}{Q}$$
>
> You will recognize this from Solow's derivation on page 312.

Assuming $\boldsymbol{\alpha + \beta = 1}$, we can substitute to get:

$$\ln Q_t = \ln A_t + \alpha \ln K_t + (1-\alpha) \ln L_t$$

Subtracting the log of labor from each side we get:

$$\ln Q_t - \ln L_t = \ln A_t + \alpha (\ln K_t - \ln L_t) .$$

This can be rewritten as:

$$\ln(Q_t / L_t) = \ln A_t + \alpha \ln(K_t / L_t) .$$

The equation above is equivalent to Solow's eqn (2a) on page 313.

Econometric specification (i.e., the population regression function)

Solow estimates this using the following (population) regression function:

(Solow's 4d) $\ln(Q/L)_t = \beta_1 + \beta_2 \ln(K/L)_t + u_t$.

Note that **(ln Q- ln L) = ln (Q/L)** and **(ln K – ln L) = ln (K/L).**

Clearly Solow is not assuming that 'technical change' does not change. So, the Y-intercept is NOT an estimate of **ln A.** Rather, he is assuming that **ln A** is stochastic, implying that the residual term (u) in eqn. 4d is the estimate of 'technical change'.

Note that the marginal product of labor, by assumption is $1-\hat{\beta}_2$.

The alternative is that, if he had computers, he could have estimated the equation $\mathbf{\ln Q = \ln A + \alpha \ln K + \beta \ln L}$ directly.

Thus, the regression function would have been

$$\ln Q_t = \beta_1 + \beta_2 \ln K_t + \beta_3 \ln L_t + u_t,$$

where instead of assuming that $\boldsymbol{\alpha + \beta = 1}$, they are estimated.

It also allows for a constant in the growth of **ln Q.** If there is a constant, it could only come from 'technical change.' So, if you think about it, in this model there are 2 components of the **ln A** term: (a) average, permanent growth of innovations and (b) a purely transitory, stochastic component (we know it is transitory because the mean of the residual term = 0).