Problems

From *Calculus*, 6th edition, by D. Hughes Hallett, G. Lozano, et al (2012)

Examples from Precalculus

1. An alternative to petroleum-based diesel fuel, biodiesel, is derived from renewable resources such as food crops, algae, and animal oils. The table shows the recent annual percent growth in US biodiesel consumption.[1]

Year	2005	2006	2007	2008	2009
% growth over previous yr	237	186.6	37.2	−11.7	7.3

(a) Find the largest time interval over which the percentage growth in the US consumption of biodiesel was an increasing function of time. Interpret what increasing means, practically speaking, in this case.
(b) Find the largest time interval over which the actual US consumption of biodiesel was an increasing function of time. Interpret what increasing means, practically speaking, in this case.

2. Hydroelectric power is electric power generated by the force of moving water. The table shows the annual percent change in hydroelectric power consumption by the US industrial sector.[2]

Year	2005	2006	2007	2008	2009
% growth over previous yr	−1.9	−10	−45.4	5.1	11

(a) According to the US Department of Energy, the US industrial sector consumed about 29 trillion BTUs of hydroelectric power in 2006. Approximately how much hydroelectric power (in trillion BTUs) did the US consume in 2007? In 2005?
(b) Graph the points showing the annual US consumption of hydroelectric power, in trillion BTUs, for the years 2004 to 2009. Label the scales on the horizontal and vertical axes.
(c) According to this data, when did the largest yearly decrease, in trillion BTUs, in the US consumption of hydroelectric power occur? What was this decrease?

3. **(a)** According to Figure 1, during what single year(s), if any, did the US consumption of wind power energy increase by at least 40%? Decrease by at least 40%?
(b) Did the US consumption of wind power energy double from 2006 to 2008?

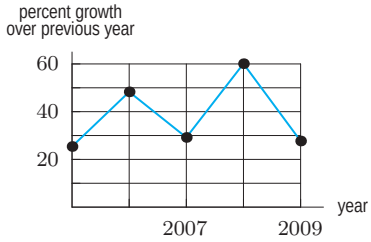

Figure 1

4. The power output, P, of a solar panel varies with the position of the sun. Let $P = 10 \sin\theta$ watts, where θ is the angle between the sun's rays and the panel, $0 \le \theta \le \pi$. On a typical summer day in Ann Arbor, Michigan, the sun rises at 6 am and sets at 8 pm and the angle is $\theta = \pi t/14$, where t is time in hours since 6 am and $0 \le t \le 14$.

(a) Write a formula for a function, $f(t)$, giving the power output of the solar panel (in watts) t hours after 6 am on a typical summer day in Ann Arbor.
(b) Graph the function $f(t)$ in part (a) for $0 \le t \le 14$.
(c) At what time is the power output greatest? What is the power output at this time?
(d) On a typical winter day in Ann Arbor, the sun rises at 8 am and sets at 5 pm. Write a formula for a function, $g(t)$, giving the power output of the solar panel (in watts) t hours after 8 am on a typical winter day.

Examples from Calculus

5. In 2011, the Greenland Ice Sheet was melting at a rate between 82 and 224 cubic km per year.[3]

(a) What derivative does this tell us about? Define the function and give units for each variable.
(b) What numerical statement can you make about the derivative? Give units.

6. Chlorofluorocarbons (CFCs) were used as propellants in spray cans until their build up in the atmosphere started destroying the ozone, which protects us from ultraviolet rays. Since the 1987 Montreal Protocol (an agreement to curb CFCs), the CFCs in the atmosphere above the US have been reduced from a high of 3200 parts per trillion

[1] http://www.eia.doe.gov/aer/renew.html. Accessed February 2011.
[2] From http://www.eia.doe.gov/aer/renew.html. Accessed February 2011.
[3] www.climate.org/topics/sea-level, accessed June 5, 2011.

(ppt) in 1994 to 2750 ppt in 2010.[4] The reduction has been approximately linear. Let $C(t)$ be the concentration of CFCs in ppt in year t.

(a) What are $C(1994)$ and $C(2010)$?
(b) Estimate $C'(1994)$ and $C'(2010)$.
(c) Assuming $C(t)$ is linear, find a formula for $C(t)$.
(d) When is $C(t)$ expected to reach 1850 ppt, the level before CFCs were introduced?
(e) If you were told that in the future, $C(t)$ would not be exactly linear, and that $C''(t) > 0$, would your answer to part (d) be too big or too small?

7. Since the 1950s, the carbon dioxide concentration in the air has been recorded at the Mauna Loa Observatory in Hawaii.[5] A graph of this data is called the Keeling Curve, after Charles Keeling, who started recording the data. With t in years since 1950, fitting functions to the data gives three models for the carbon dioxide concentration in parts per million (ppm):

$$f(t) = 303 + 1.3t$$
$$g(t) = 304e^{0.0038t}$$
$$h(t) = 0.0135t^2 + 0.5133t + 310.5.$$

(a) What family of function is used in each model?
(b) Find the rate of change of carbon dioxide in 2010 in each of the three models. Give units.
(c) Arrange the three models in increasing order of the rates of change they give for 2010. (Which model predicts the largest rate of change in 2010? Which predicts the smallest?)
(d) Consider the same three models for all positive time t. Will the ordering in part (c) remain the same for all t? If not, how will it change?

Problems 8–9 concern hybrid cars such as the Toyota Prius that are powered by a gas-engine, electric-motor combination, but can also function in Electric-Vehicle (EV) only mode. Figure 2 shows the velocity, v, of a 2010 Prius Plug-in Hybrid Prototype operating in normal hybrid mode and EV only mode, respectively, while accelerating from a stoplight.[6]

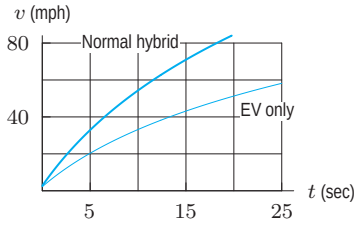

Figure 2

8. Could the car travel half a mile in EV only mode during the first 25 seconds of movement?

9. Assume two identical cars, one running in normal hybrid mode and one running in EV only mode, accelerate together in a straight path from a stoplight. Approximately how far apart are the cars after 15 seconds?

Problems 10–12 refer to a May 2, 2010, article:[7]

> "The crisis began around 10 am yesterday when a 10-foot wide pipe in Weston sprang a leak, which worsened throughout the afternoon and eventually cut off Greater Boston from the Quabbin Reservoir, where most of its water supply is stored... Before water was shut off to the ruptured pipe [at 6:40 pm], brown water had been roaring from a massive crater [at a rate of] 8 million gallons an hour rushing into the nearby Charles River."

Let $r(t)$ be the rate in gallons/hr that water flowed from the pipe t hours after it sprang its leak.

10. Which is larger: $\int_0^2 r(t)\,dt$ or $\int_2^4 r(t)\,dt$?

11. Which is larger: $\int_0^4 r(t)\,dt$ or $4r(4)$?

12. Give a reasonable overestimate of $\int_0^8 r(t)\,dt$.

[4] www.esrl.noaa.gov/gmd/odgi
[5] www.esrl.hoaa.gov/gmd/ccgg
[6] www.motortrend.com/, accessed May 2011.
[7] "A catastrophic rupture hits region's water system" in *The Boston Globe* May 2, 2010.