How Long is a Dinosaur?
How Big is an Atom?
An Introduction to Practical Analytic Geometry

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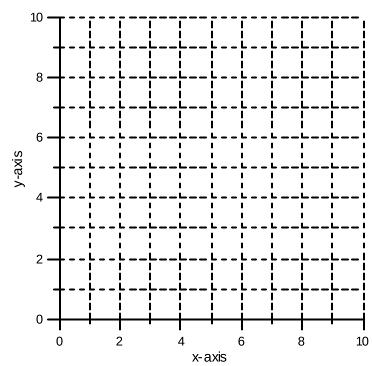
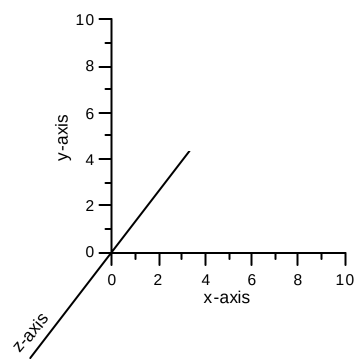
Geometry is central to much understanding of the natural world, and a knack for spatial visualization is critical in many aspects of the earth sciences. We describe fossils and unit cells by their sizes and shapes, we calculate volumes and angles, we make maps and cross sections.

Coordinate Systems

There are about a jillion coordinate systems available, some very intricate and elegant, but for the most part we'll stick to the simplest and oldest one here, the Cartesian system in two and three dimensions. (The system is named after Rene Descartes, 1596 – 1650, a French aristocrat who pretty much invented analytic geometry.)

The fundamental notion is that we can specify the location of a point in space by measuring its distance from a designated reference point, or *origin*, along pre-defined directions, or *axes*. For location on a plane, we need two axes, for solids we need three.

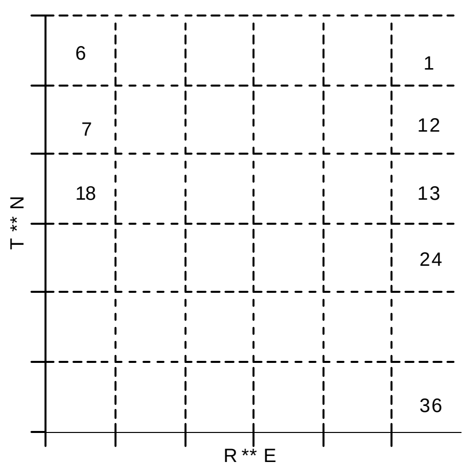




Two- and three-dimensional Cartesian coordinate systems (though shown only as a 2-D representation of same).

Township & range system of land survey

As a very important aside, you should learn about the standard means of land survey in the US of A. When the old timers were parceling out land for sale in the newly formed US, they adopted a system devised by, among others, George Washington. It relies on the notion of a Cartesian coordinate system with east-west and north-south principle axes, referred to as the Baseline and Meridian, respectively. The axes are measured off in six mile intervals, called townships in the north-south direction and ranges to the east and west. Each 36 square mile block so defined is referred to as a township (overuse of the word, eh?). The township blocks are divided up into 36 one-mile squares, called "sections" and numbered in the standard system sketched below.



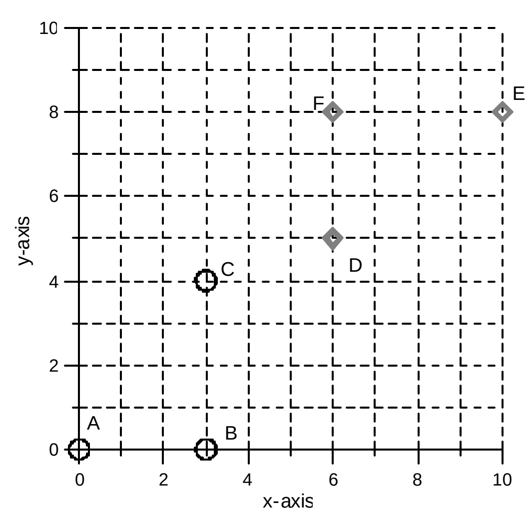
All of the country except for the eastern seaboard, Kentucky, Tennessee, and Texas (but including Florida) has been surveyed using this system. The attached map shows the standard baselines and meridians currently in use in the US.

Distances between Points

On a plane, the distance between any two points may be readily found by judicious use of two principles: Euclid's axiom that the shortest distance between two points is a straight line, and the Pythagorean Theorem.

The sketch at the right illustrates the concept using points A through F. The distances between points A and B, or between D and F, is easily seen to be 3 units. There's no Pythagoras in view.

On the other hand, the distance from A to C requires more smarts to figure out. We can always measure it directly with a ruler (and then multiply by an appropriate factor to fit the scale of the plot); sometimes this is the most practical way to go. But since we already have the coordinates



of the points at hand, we can invoke Pythagoras and calculate the distance as $r = \sqrt{x^2 + y^2}$ Overall, this latter method is more accurate, robust, and useful. Notice that there is another triangle, DEF, exactly like the first except that it doesn't have an apex at the origin. To calculate the length of its hypotenuse, DE, we would use $r = \sqrt{(x_D - x_E)^2 + (y_D - y_E)^2}$, where the order of the subscripts doesn't matter because of the squaring. The two terms under the square root sign are just the differences between the points in the principle *x*- and *y*- directions, which are then squared, as above.

The concept of a Pythagorean length can be extended to more than three dimensions, even though we can't visualize them as spatial. The generalized form of the Pythagorean equation looks like $r = \sqrt{(x_{1,1} - x_{1,2})^2 + (x_{2,1} - x_{2,2})^2 + ... + (x_{i,1} - x_{i,2})^2}$ As we shall see later in the course, this can be very powerful in data interpretation.

For now, let's formalize a few necessary basic concepts. To that end, consider the following illustrative problems.

Vector Representation:

A convenient way to specify the coordinates of a point is by putting them into an ordered array, *e.g.*, (x,y) or (x,y,z). In this system, each *component* of the vector is an *element* of the array. In fact, we generally find it more convenient to refer to the elements, and thus the components, by subscripts rather than by individual names, thus we see expressions like $x = x_1, x_2, x_3$ for the location of point X, or $rad = m_1, m_2, m_3$ for the radius vector to the moon. This common practice is precisely why a matrix with only one row or one column is called a vector.

Problem 1: First Build a Tool Write a MatLab function called *pythag* that calculates the Pythagorean length of a vector having an arbitrarily number of components.

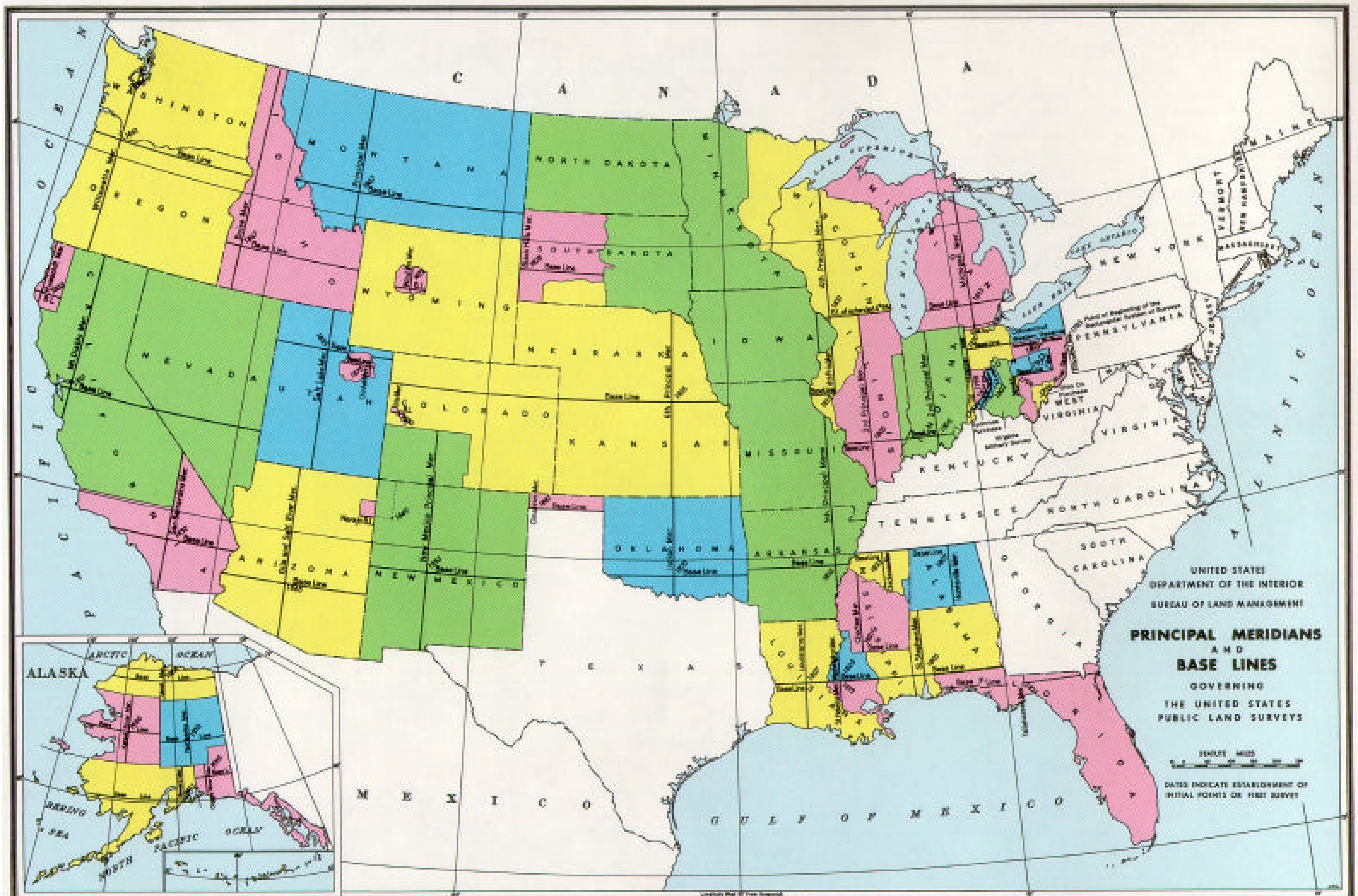
What we're after here is a bit of code to do the desired calculation and which is reusable in later endeavors, thus it will be a *function*. Once you get it worked out, you will be able to reference *pythag* in other programs just by having it present in your workspace. Refer to the MatLab Help files on functions. Note that the standard MatLab function *length* could be very useful for determining the limit on a *for* loop that does the heavy lifting. (Note that many handheld calculators have a 2-D version available.)

Problem 2: The Dinosaur Part A group of underground uranium miners working in the Morrison formation of southeastern Utah encountered a *T. rex* skull on the 180-level (*i.e.*, 180 feet below surface) at a distance of 95 feet along the drift due east from the main shaft. A few days later another crew working on the 200-level drills into what looks like pieces of *T. rex* tailbone 10 feet south of a point 80 feet along the drift. If these represent the ends of the same critter, how long is the dinosaur?

Problem 3: Another View of Dinosaurs Miners from two different outfits stop fighting long enough to discover that they have each discovered ends of dinosaurs in their day jobs. After some negotiation, they realize that their respective tunnels are close together, and they may be dealing with opposite ends of the same fossil. The team from Big Bang Uranium Corp found tail bones 125 feet from their main shaft on the 75-level, which bears S30E. The guys from Litigation Resources ran into skull fragments 90 feet from their shaft on the 200-level, which runs N40E. The headframes of the two mines are 190 feet apart, with the BBUC N13W of the LR mine, which is 50 feet higher. Once again, how long is the total skeleton, if they are bits of the same beast?

Problem 4: On a different scale Consider a crystal structure consisting of a cubic array of chlorine atoms with an alkali cation situated at the center of the cube (this might be halite, NaCl, the first crystal structure ever worked out, by W.H. Bragg). Given the radii of the alkalis and of chlorine, what would be the range of compounds that might be formed in this system?

Ionic radii: Cl^- 1.81 Li 0.60 Na 0.95 K 1.33 Rb 1.48 Cs 1.69 Ag^+ 1.26



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