Math mechanics - can you crank out a correct answer?

This is a list of mathematical concepts that incoming graduate students should be able to use or at least define. Many students will have had exposure to these concepts prior to arriving at college, but maybe not to the extent that they can actually use them. The list was developed to encompass math concepts that we use in our own research, and hence would want our students to be able to use as well. This list should be considered a work in progress, and feedback and suggestions are encouraged. Items in **bold** may be important to include as well.

Basics
1) graphing, graph interpretation
2) unit conversion
3) dimensional analysis
4) back-of-the-envelope, order of magnitude calculations
5) substitution of variables
6) solving systems of equations (link to linear algebra)

Functions
1) Dependent vs. independent variables
2) Separation of variables
3) Types of functions (linear, power, exponential, logarithmic)
4) Periodic (trig) functions
5) **Time series analysis**

Multi-variable Analysis
1) trigonometry
2) vectors
3) directional derivatives,
4) gradient, slope
5) matrices
6) linear algebra
7) sensitivity analysis
8) **eigenvalues**

Statistics and Probability
1) Error analysis (mean, median, std deviation, confidence interval)
2) regression analysis (R^2, χ^2)
3) Conditional probability
4) Accuracy analysis
5) Probability distributions
6) **Signal processing/pattern recognition**
7) **Bayesian stats**

Calculus
1) Ratio
2) Rate

3) Sum and integral
4) Derivative and Partial derivative

Uncertainty:

Students should appreciate that science is intrinsically uncertain. It is important that they be comfortable with the notion that a deterministic description of a state or a process does not exist, and this does not render the knowledge useless. Sources of uncertainty are numerous, and students should be able to identify and quantify sources of error such as experimental design, measurement error, propagation of error, model inadequacy, etc. Students should be able to quantify uncertainty and demonstrate an ability to use the uncertainty estimates to assess the quality of a solution.

Logical analysis

Students should be able to take a problem, devise strategies for addressing the problem, implement strategies with appropriate tools and skills, and work toward resolution of the problem and understanding the results. These are some of the skills that could be used for various parts of logical analysis

a) Recognize the type of problem, e.g.,
   - Stating the problem
   - Breaking the problem into a series of logical steps.
   - Determining what you know and don't know

b) Identify the appropriate approach and analytical tools, e.g.,
   - Deductive analysis
   - Inductive analysis
   - Making appropriate assumptions
   - Modeling

c) Estimate possible solutions, e.g.,
   - Understanding quality of data/model
   - Performing back-of-the-envelope calculations

d) Apply tools to solve the problem, e.g.,
   - Programming
   - Further inductive and/or deductive analysis

e) Logically assess the validity of an answer, e.g.,
   - Evaluating the reasonableness of a solution
   - Determining if there is something new
   - Assessing consistency with assumption
   - Evaluating uncertainty
   - Understanding if there are broader scientific implications

An example: A leaking underground storage tank, perhaps containing toxins

a) Is this really a problem? What is leaking? How fast is it spreading? What is its toxicity level? What are relevant environmental conditions? ….

b) Identify possible modeling: MODFLOW, develop own model (thermodynamics, fluid flow, chemical degradation…)
1. Identify possible empirical data: collecting cores, water quality
2. Identify where to sample and other data needed (for modeling. sampling strategy…)

c) Initial estimates of contaminant spreading (consequences, timing, remediation…)

d) Build and run models, take the samples

e) What does this mean for local land use? What are the costs/benefits? What is communicated to the general public? What should be published and where?

Ability to learn independently

Students need to gain the confidence to understand that resources are available that enable them to solve problems. Students should practice using outside resources (e.g, books, web, journals) to gain the tools and methods to solve problems. Students need to gain confidence that they can expand their mathematical skill sets on their own. Transferring skills to new problems underlies the whole research process. It can involve solving quantitative problems with mathematical tools that are not traditionally used. We would like students to pick up tools and techniques from textbooks and modify or combine them appropriately to solve a new problem.