Fourier Optics and Diffraction

Diffraction and Fourier optics are very closely related. You may recall the Fourier transform of the aperture gives the Fraunhofer diffraction pattern. In class, we discussed how we can use a lens to perform a Fourier transform. In this lab, we will investigate the connection between Fourier Optics and Fraunhofer diffraction, we will look at the diffraction pattern of various objects, and we will see how we can use diffraction to isolate spatial modes of the laser.

We learned that a simple lens can be used to take a Fourier transform. Consider the following set-up below:

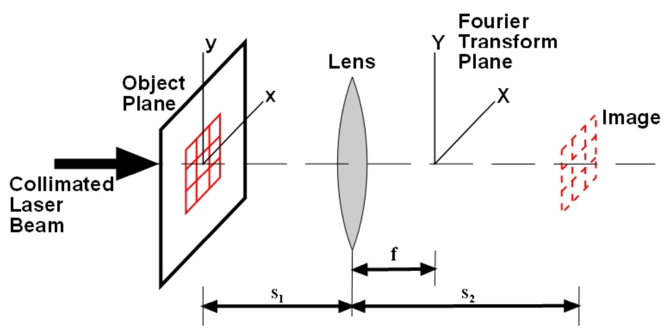


Figure 1. Lens as a Fourier Transform Set-up**.**

In Figure 1, we see a collimated laser beam illuminates a black-and-white transparency, which serves as the object. The Fourier transform appears a distance f behind the lens. Using the lens equation, we can also observe an image of the object a distance s_2 away from the lens if the object is located at s_1.

If $E_0(x,y)$ is the electric field distribution at the object plane, which is described by the spatial coordinates x and y, then the electric field in the Fourier Plane $E_{FT}(X,Y)$ is proportional to the Fourier transform of the electric field at the object plane:

$$E_{FT}(X,Y) \propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_0(x,y)\exp\left[-i2\pi\left(\frac{X}{f\lambda}\right)x\right]\exp\left[-i2\pi\left(\frac{Y}{f\lambda}\right)y\right]dxdy \qquad [6]$$

where X and Y are the spatial coordinate of the Fourier plane.

You would naturally say the physical interpretation of the coordinates X and Y is that they represent displacement. However, a closer look reveals another interpretation. The quantities $(X/f\lambda)$ and $(Y/f\lambda)$ have the dimension of spatial frequency (cycles/meter or meter^{-1}). These quantities correspond to the spatial frequency coordinates of the objects Fourier transform (i.e., the coordinates in Fourier Space).

To give a concrete example, if an object contains a periodic feature, such as a grating of spatial frequency $(X_0/f\lambda)$ cycles/meter, there will be a "dot" in the Fourier plane at a distance X_0 from the origin. Actually two dots, one each at $\pm X_0$. (Remember that in the Fourier plane a sine wave corresponds to two delta functions.)

Goal: Spatially filter a laser beam and study the Fraunhofer diffraction pattern of various apertures.

Equipment:

1 – Laser
1 – Periscope (Optional)
4 – Mirrors and associated optomechanics
2 – Pinholes (25um and 50um)
1 – Spatial filter set-up (microscope objective, pinhole, mount)
2 – Lenses
Various aperture shapes and associated optomechanics

Part 1: Spatial Filtering

Theory

Low-pass spatial filtering is a technique frequently used to improve the quality of a laser beam. Just as different frequencies of sine waves are added together to create an arbitrary function, similarly the output of a laser can be described by the sum of different laser modes. These other laser modes are called high spatial frequency laser modes or just “higher-order modes”. A poor quality laser will have an irregularly shaped output beam because of these higher-order modes. The idea behind spatial filtering is that the higher order modes are spread out in angle. The higher the mode, the angular frequency which results in more spatial separation in the Fourier plane. The pure TEM_{00} mode, which roughly corresponds to a plane wave, is selected from the central region using an aperture. Since spatial frequency increases with distance from the central region, blocking everything except for the central frequencies we are allowing only low spatial frequencies to pass.

I could try to explain this further, but there is an attached handout, which does a great job and has some handy diagrams. Stop and read that handout now.

Now for some math…The diffraction limited spot size, *a*, generated by an objective is given by the relation:

$$a = \frac{1.22\lambda f}{d_{in}} \quad [1]$$

where λ is the wavelength, f is the focal length of the objective and d_{in} is the beam diameter at the input of the objective. The beam from the laser is Gaussian with a beam waist, w_0 and a divergence, θ, if the objective is a distance L away from the laser head, the input beam diameter is given by the relation:

$$d_{in} = \sqrt{{w_0}^2 + 4\theta^2 L^2} \quad [2]$$

From the laser parameter lab, you should be able to calculated d_{in}.
There are two pieces of information etched into the side of the objective: the NA and the magnification *M*. If there isn’t an infinity symbol etched into the side as well (indicates the objective is infinity corrected), the objective was meant to operate with a microscope tube of length 160 mm. The focal length of the objective is given by:

$$f = \frac{L_{tube}}{M} = \frac{160mm}{M} \quad [3]$$

Once the objective is chosen, the pinhole is determined by calculating the diffraction limited spot size and choosing a pinhole that is 1.5x the diffraction limited spot size. The factor of 1.5 optimizes the energy passing through the pinhole while optimally filtering the beam.

Aligning the Spatial Filter

Aligning a spatial filter is a very delicate business. In fact, you will use a special mount for this purpose that allows the appropriate degrees of freedom (translation in x/y/z and tilt in x-z and y/z) while keeping the pinhole and objective aligned with respect to each other. Since pinhole is so incredibly small, it is extremely important the beam travel along the optic axis of the objective. Therefore, the mount has an iris that helps you make sure the laser is centered on the objective. The iris, combined with the tricks we learned in earlier labs about using the forward beam and the back reflections, are important tools to accomplish spatial filtering.

So that you have time to do other parts of the lab, I have done most of the alignment of the spatial filter for you. You may have to do some minor tweaking to re-optimize it. Below are some tricks of the trade to help you. If you find you are making significant adjustments, stop what you are doing and get me!

1. Start with a larger size pinhole than you actually plan to use if the actual pinhole you want is less than 25 um.

2. Monitor the reflection off of the objective lens onto the laser head while making sure the light traveling though the objective is traveling along the optic axis. Look for the back reflection off of the objective on the iris you placed just after the laser head. If the reflected spot is far from the laser aperture, adjust the spatial filter orientation. The iris on the front of the spatial filter can help in making sure the input beam in centered on the objective aperture.

3. Place a white card after the spatial filter and adjust the X-Y position of the pinhole until a faint spot is incident on the card. If you don't initially see a spot on the card, go to step 2. As soon as you see any light on the card, use it as your feedback method.

4. Turn the room lights off and look at the front of the pinhole (OBLIQLY – NEVER PUT YOUR HEAD AT TABLE LEVEL). You should see a dim red gleam from the pinhole. Adjust the pinhole position to maximize the intensity.

5. Rotate the knurled nut to bring the pinhole closer to the objective and adjust the X-Y position of the pinhole to maximize the intensity on the white card.

6. Repeat the previous step until the maximum throughput is obtained. This is indicated by a bright "circular blob" of light surrounded by a symmetric pattern at the output. A slight movement of the X-Y position of the pinhole results in the complete disappearance of the diffraction pattern as well. An example is illustrated in Figure 2 below.

Figure 2. Output of the spatially filtered HeNe laser beam. The spatial filter filter was 10 um and the magnification of the objective was 20x (f = 8mm, NA=0.4). The diffraction limited spot size after the objective was 12.36 um, so the output power in the central lobe was 22.8% of the input power. The pinhole in this photo was chosen to be somewhat small to show the diffraction rings. A properly filtered beam is a smooth, nearly Gaussian profile with no visible features. Rings like those shown can occur with a properly-size pinhole when the pinhole is not at the focus of the objective. (Courtesy of Basic Skills Lab from Dr. Steve Cundiff and Edward McKenna at the University of Colorado).

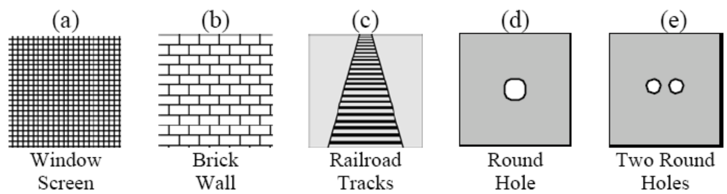
7. Replace the larger um pinhole with a smaller pinhole. Then repeat steps 4-6.

Choose a lens to collimate the spatially filtered beam so that the beam is at least 0.75 inches in diameter. Show any diagrams and/or calculations used to choose the focal length. Use the shear plate collimator to check your collimation.

Now that the laser has been spatially filtered and collimated, we will now investigate Fourier Transforms.

Part 2: Fourier Transforms

Here is a warm up exercise before you start this activity. Using your knowledge of Fourier Transforms, discuss the diffraction pattern associated with the objects below with your lab partner. Assume the objects are two-toned – either entirely transmitting or opaque.



1. Obtain a long focal length lenses; say greater than 20 cm. (Why do you want the focal length to be long? This will be your Fourier transform lens.
2. Obtain several patterns/objects that will serve as objects in your Fourier optics set-up.
3. Make predictions regarding the Fraunhofer diffraction pattern you expect to see.
   a. Sketch the pattern
   b. What do you expect to happen when you translate the object?
   c. What do you expect to happen when you rotate the object about an axis perpendicular to its plane?
4. Check your predictions regarding your patterns.
5. Determine characteristic parameters about one or more of your objects (e.g. slit width, grid spacing, etc.)
6. Thought experiment: If you took the diffraction pattern that you obtained, turned it into an object and used it in your Fourier transform set-up, would you recover your original object?

Resources

Pedrotti Chapter 11, Section 21-1, Section 27-7
Saleh Chapter 4
Hecht Sections 10.1-2 Chapter 11, pgs 592-593