**Elementary Statistics—Sampling-Distribution Exercises**

Work as a group (not as individuals sitting in close proximity). *Every member of the group must give a “thumbs up” before the group moves to the next problem.* These exercises ask for more and more explanation. You need not turn in your answers, but it’s important that you sketch your solutions (to refer back to when studying). Also, it’s more important that you have detailed, thorough discussions than that you record those discussions exactly. That is, spend time talking through all these questions in detail (including a careful explanation of your answers), and your “solutions” can be simply a sketch of those detailed discussions. (I want to ensure you get through all the problems.)

1. Consider a population of just 10 college students. These ten students all have Facebook accounts. The number of Facebook “friends” for each of these college students: 100, 110, 90, 70, 60, 100, 90, 90, 150, 130.

This average number of friends for this small population is $μ=99$. Also, the distribution of the population is shown in the dotplot below.



As we discussed in class, the sample average (based on a random sample) is a good estimator for the population mean (it’s an unbiased estimator, $μ\_{\overbar{x}}=μ$ ,and its variability, $σ\_{\overbar{x}}=\frac{σ}{\sqrt{n}}$ , decreases as the sample size increases). *What is the sampling distribution of the sample average, based on samples of size 3?* To investigate this, 20 different samples (of size 3) were randomly selected from the population of 10 friends. For each sample, the sample average, $\overbar{x}$, is computed. (For example, one of the random samples consisted of the numbers 70, 100, and 150. For this particular sample the $\overbar{x}$ value is 106.7.) These 20 $\overbar{x}$ values are shown in the dotplot below. The graph shown below is an estimate of the sampling distribution of $\overbar{x}$ (the actual sampling distribution would show the $\overbar{x}$ from *all* possible samples of size 3 from this population).



* 1. Consider the two dotplots above. How exactly are these dotplots different? For example, i) what do the dots represent in each plot?, and ii) how are the shapes of the distributions different? (You can be quite brief with your answers. This is the warm-up exercise.)
1. Consider four different distributions: *1)* The distribution of the number of Facebook friends for a *population* of 1000 college students, *2)* the distribution of Facebook-friend values for *one sample* of size 200 (taken from the population described in *1*), *3)* the distribution of number-of-Facebook-friend *averages* for 1000 different samples (all of size n=10) from the population mentioned in *1*, and 4) the distribution of number-of-Facebook-friend *averages* for 1000 different samples (all of size n=50) from the population mentioned in *1*.

Four frequency histograms are shown below. Match each histogram with exactly one of the distributions listed above. Thoughtfully defend your answer for *every* match (not just “it was the one leftover”).



1. Suppose the distribution of Facebook friends for all college students is strongly skewed to the higher values with mean 120 and standard deviation 73. (Use this information for parts *a-c* of this problem.)
	1. Based on the methods of analysis you’ve accumulated so far, state explicitly why you *cannot* determine the probability that a randomly selected college student has more than 100 friends.
	2. Now, state exactly why you *can* determine the probability that the *average* number of friends (for a random sample of size 50) is larger than 100.
	3. Determine the probability asked for in part b. As part of your solution, draw an appropriate, well-labeled picture, and for all your calculations *explain exactly what you are doing* (e.g., if you use a formula, explain exactly how it helps you solve the problem and why exactly you “plug in” certain numbers into the equation). Table A is included on the last page.
2. Disregard the information from the previous question. The Facebook CEO claims the average number of Facebook friends for the population of all college students is 150 (and the population standard deviation is 73). Using your statistical knowledge, you effectively take a random sample of 50 college students on Facebook and record the number of “friends” for each of them. The sample average for these 50 students is $\overbar{x}=115$ Facebook friends.
	1. **For the next two parts of this problem, ask and answer the following questions:** **1)** What exactly are you doing (can you describe it precisely)?, **2)** why are you doing it (how does it fit into the solution)?, and **3)** how does it help you (what will you do with the outcome you obtain)?
	2. Assuming the Facebook CEO’s claim $(μ=150)$ is true, determine the probability of observing this sample average (115) or a smaller sample average of friends. (Include an appropriate, well-labeled picture as part of your answer.)
	3. Does the probability you determined in part b make you doubt the population average claimed by the CEO? Why or why not?
3. Disregard all the previous information given. Now suppose the population of the number of Facebook friends is approximately normal (this is probably unrealistic, but let’s suppose it’s the case—perhaps it’s not completely unrealistic for the population of college students). Furthermore, suppose the average number of Facebook friends for the population of college students is 180 and the standard deviation is 38.
	1. Look back at your answer to problem 3, part a. What’s different about this problem that allows you to determine the probability that a randomly selected college student has more than 100 friends?
	2. We know both the population standard deviation ($σ=38$) and the standard deviation of the $\overbar{x}$ sampling distribution ($σ\_{\overbar{x}}=\frac{σ}{\sqrt{n}}=\frac{38}{\sqrt{n}}$). And we know that the individual values and the average values follow approximately normal distributions (just with different standard deviations). For each of the following scenarios, give the standard deviation that would be used in the z-value. (*You needn’t solve the problem, just provide the standard deviation appropriate to the problem.*)
		1. For a random sample of 100 college students, what’s the chance the sample average is more than 175 Facebook friends?
		2. For a randomly selected college student, what’s the chance the student has fewer than 150 Facebook friends?
		3. A college student is selected at random and she has 250 Facebook friends. Is this surprising?
		4. A random sample of 300 college students is taken, and the sample average number of Facebook friends is 170. Is this sample average surprising?