Paper No. 60-11

Modeling U-Series Concordia/Discordia Using STELLA

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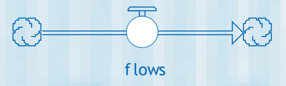
Introduction

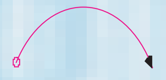
U-Series dating techniques are widely used to determine the absolute ages of some of Earth's oldest rocks, but the concordia/discordia diagram can be quite difficult for students to grasp. In particular, the fact that differing amounts of lead and uranium loss from minerals such as zircon can be used to determine not only the original formation age, but also the time of metamorphism of a rock like granite, is a challenging concept. Making use of previous workers' web-published excercises on radiometric decay, I have produced a STELLA-based lab exercise to develop students' understanding of this important chronologic technique. Students create models of the two isotopic decay systems, ^{238}U --> ^{206}Pb and ^{235}U --> ^{207}Pb, and run these models for 4.5 billion years to create the concordia diagram. They then carry out experiments in which they "add" or "remove" varying amounts of lead or uranium in simulation of metamorphism. The uranium-lead ratios at the end of the simulation allow the discordia line to be plotted on top of the concordia diagram and the ages of original crystallization and metamorphism to be determined from the points of intersection of the two lines. In the course of the lab, students are introduced to the concepts of exponential decay and secular equilibrium as well as modeling concepts such as the creation of if-then statements.

STELLA

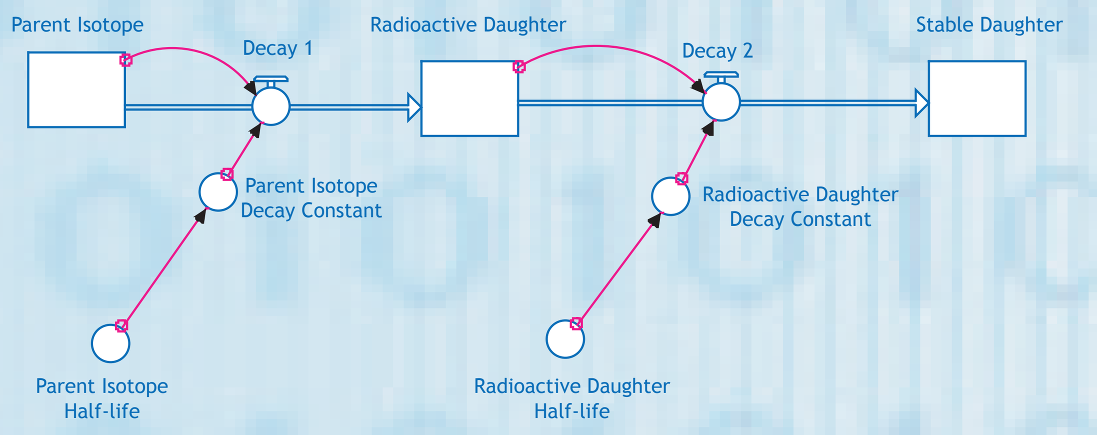
Structural Thinking Experimental Learning Laboratory with Animation - icon-based dynamical systems modeling tool developed by High Performance Systems, Inc.

- ★ Boxes represent reservoirs 

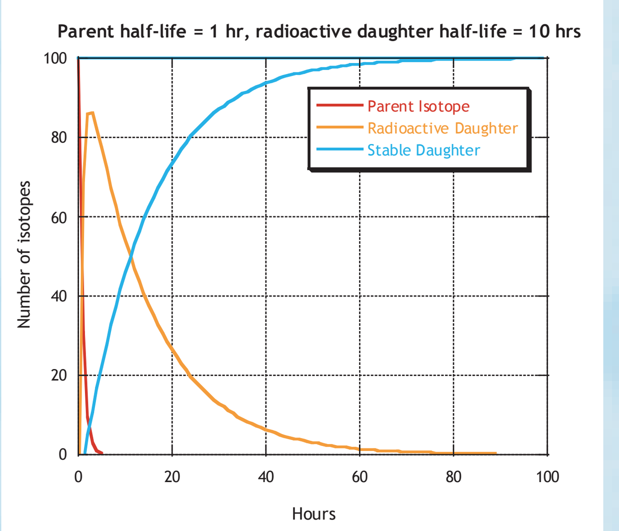
- ★ Arrows represent flows into and out of reservoirs 

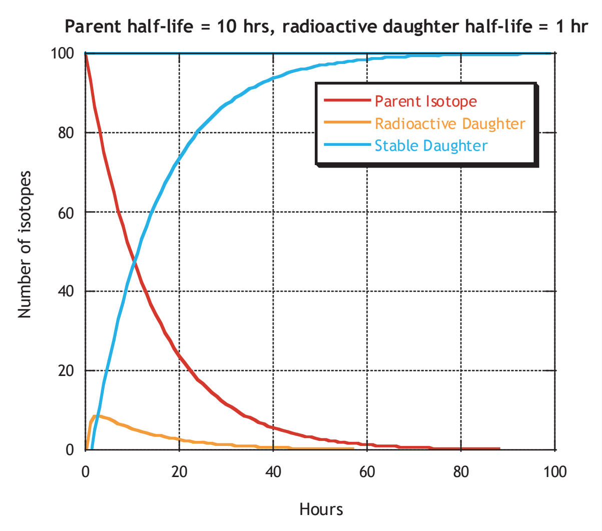
- ★ Dependencies of variables are represented with pink linking arrows 
- ★ Converters, hold values of constants and equations. 

1.



STELLA model of a three-isotope decay series (above) and experiments with different half-lives for the radioactive parent and daughter isotopes (below). Decay flows = the number of isotopes remaining at time t * the decay constant. Decay constants (λ) calculated from half-lives by λ = ln(2)/half-life.





2.

$$-\frac{dN_1}{dt} = \lambda_1 N_1 \qquad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

solution: $N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1^0 e^{-\lambda_1 t}$

but, $N_1 = N_1^0 e^{-\lambda_1 t}$

so, $N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1$

rearranging: $\frac{N_1}{N_2} = \frac{\lambda_2 - \lambda_1}{\lambda_1}$

if $\lambda_1 << \lambda_2$, then $\lambda_2 - \lambda_1 = \lambda_2$, so

$\frac{N_1}{N_2} = \frac{\lambda_2}{\lambda_1}$

or, $\lambda_1 N_1 = \lambda_2 N_2$ and $N_2 = \frac{\lambda_1}{\lambda_2} N_1$

$D^* = N_1^0 - N_1 - N_2$ but since $\lambda_1 << \lambda_2$, $N_2 = 0$

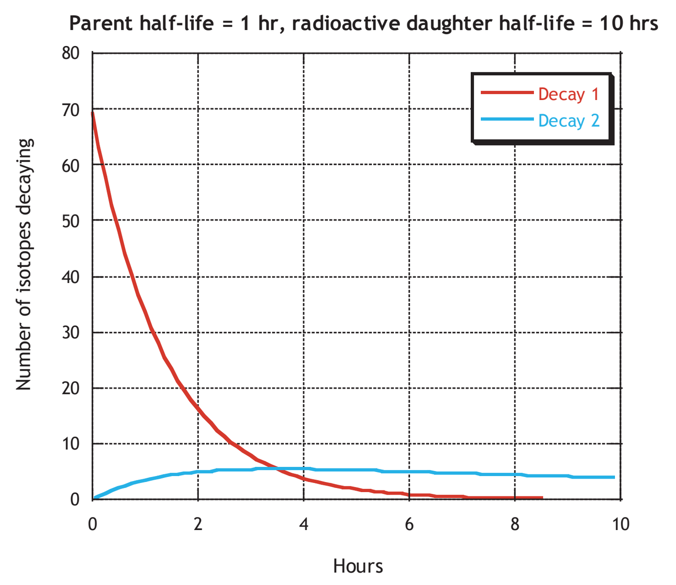
$D^* = N_1 e^{\lambda_1 t} - N_1$

Secular Equilibrium

The complicated Uranium decay series (4., right) can be simplified due to the fact that both series exhibit secular equilibrium. This condition is met when the half-life of a radioactive parent isotope at the beginning of a decay series is much longer than the half-lives of any intermediate daughter isotopes. The mathematics to the left illustrate this point for a simple three-isotope system in which a radioactive parent (N_1) decays to a radioactive daughter (N_2), which in turn decays to a stable daughter (D^*). The change in the population of parent isotopes with time ($-dN_1/dt$) is dependent on the decay constant of the parent isotope (λ_1) and the number of parents remaining at time t. For the radioactive daughter, N_2, the change in its population (dN_2/dt) is a function of the production of new daughters from the decay of radioactive parents ($\lambda_1 N_1$) minus the loss of radioactive daughters due to decay to the stable daughter ($\lambda_2 N_2$). The solution to this pair of differential equations gives the curious result that $\lambda_1 N_1 = \lambda_2 N_2$ = a constant. This result can be generalized to a system with n species, in which case, $\lambda_1 N_1 = \lambda_2 N_2 = \ldots = \lambda_n N_n$.

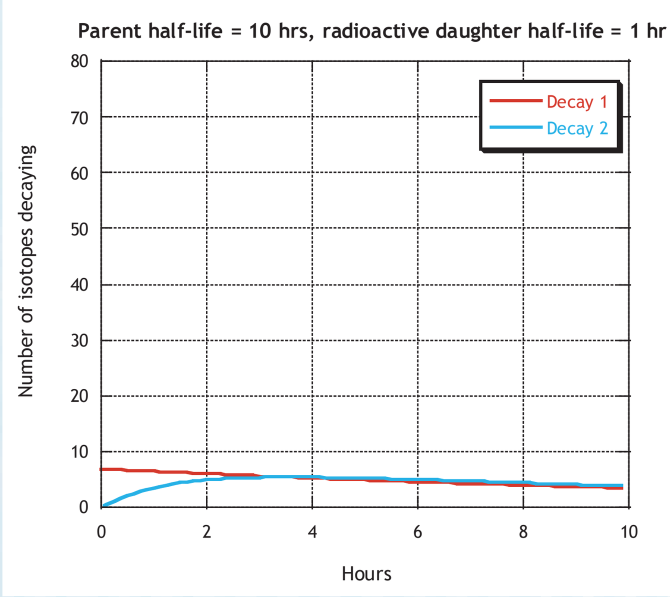
The number of stable daughters at any time (D^*), is the difference between the initial number of radioactive parents (N_1^0) and the current number of radioactive parents (N_1) and radioactive daughters (N_2). $N_2 = (\lambda_1/\lambda_2)*N_1$ and $\lambda_1 << \lambda_2$, N_2 is effectively zero and can be ignored. Thus, when the conditions of secular equilibrium are met, the decay series can be treated as though the radioactive parent isotopes decay directly to stable daughters.

3.

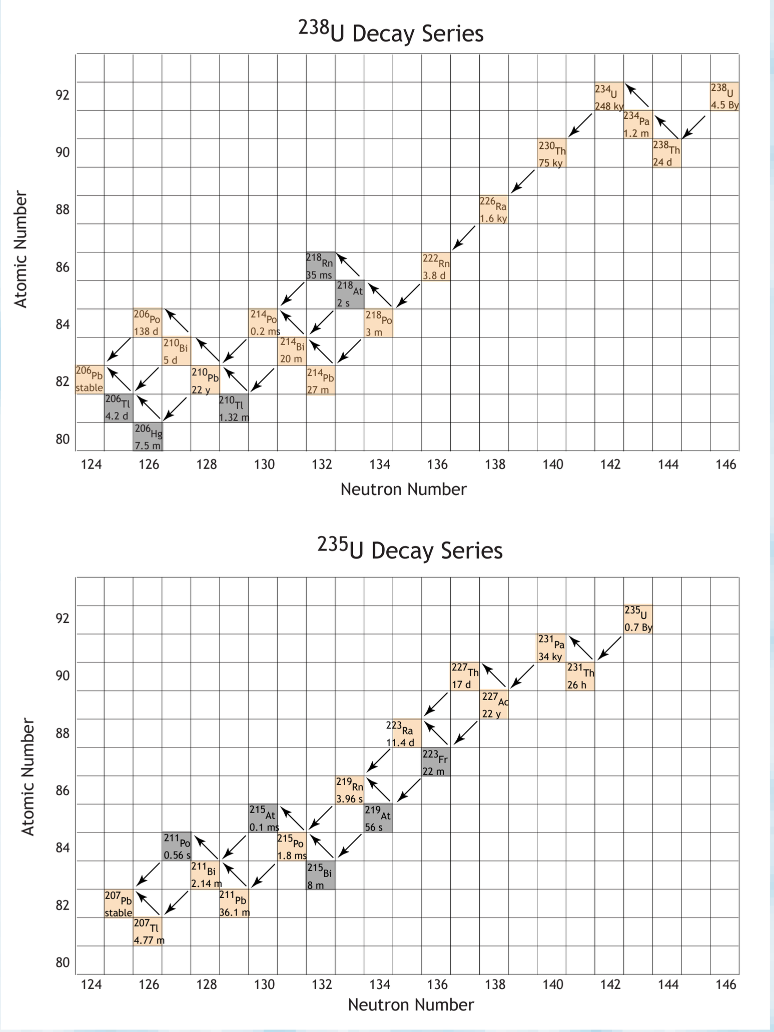


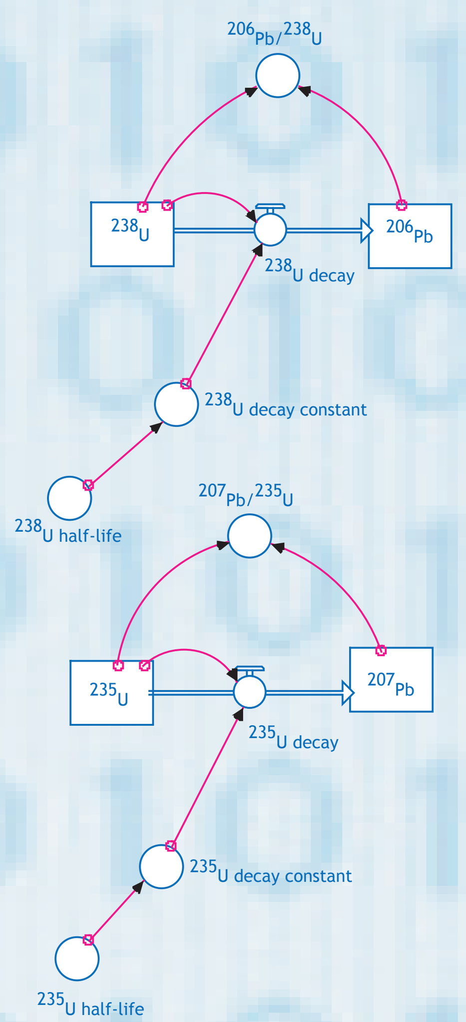
Secular equilibrium experiments. In the example above, the half-life of the radioactive daughter exceeds that of the parent such that secular equilibrium is not achieved. Note that the curves representing the number of isotopes decaying do not converge. In other words, $\lambda_1 N_1 = \lambda_2 N_2$.

In the example below, the parent half-life greatly exceeds that of the radioactive daughter, such that $\lambda_1 N_1 = \lambda_2 N_2$ within a short period of time. Secular equilibrium!



4.

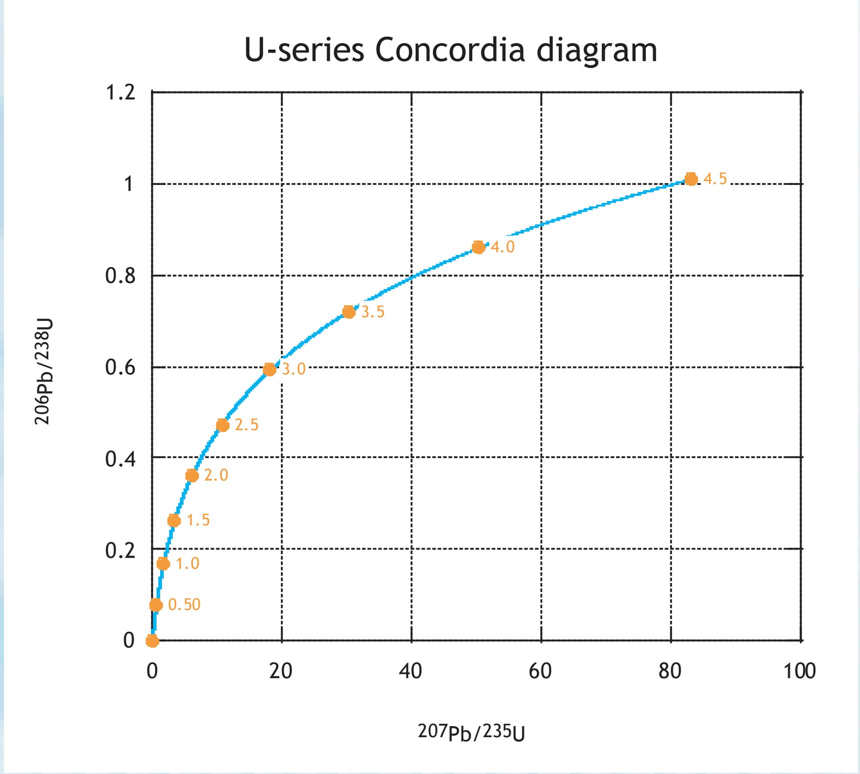




Decay series for ^{238}U and ^{235}U (above left). Primary decay series shown in orange. Half-lives of isotopes shown below isotope name. By = billion years, ky = kiloyears, d = days, h = hours, m = minutes, s = seconds, ms = milliseconds. Note that in all cases, the half-life of the initial decay is much longer than that of any of the intermediate daughter products such that secular equilibrium can be assumed. STELLA representations of U-series decay (above right), making use of secular equilibrium simplification.

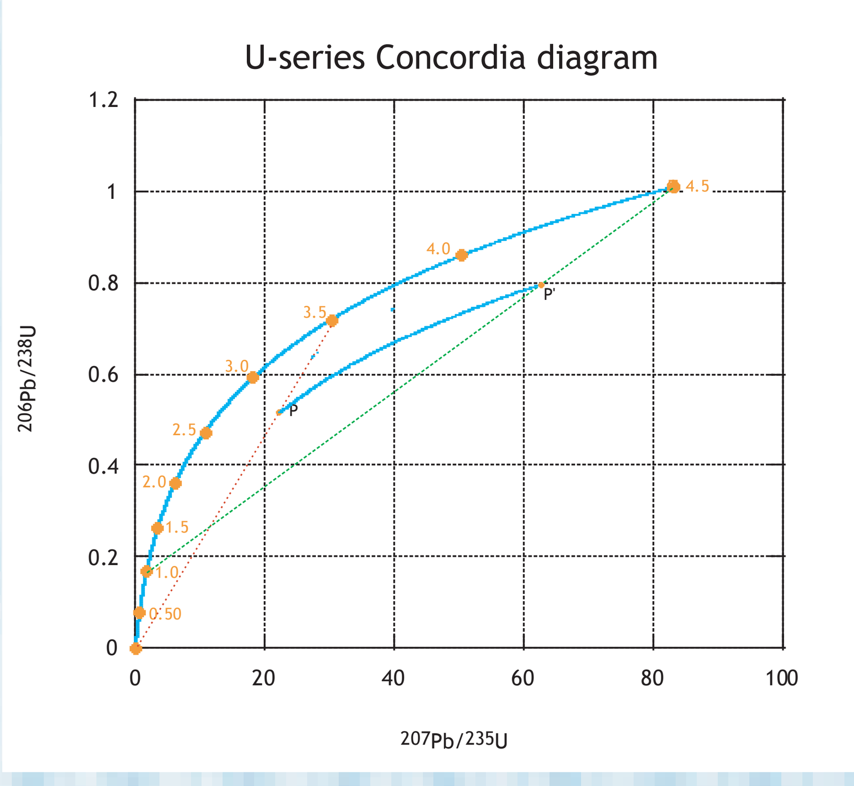
5.

Resulting Concordia diagram (below) when both Uranium reservoirs begin with 1 million atoms, both Lead reservoirs start out empty, and the model is run for 4.5 billion years (age of the Earth). Orange points represent 0.5 billion year time increments. The longer half-life for ^{238}U relative to ^{235}U is reflected in the x and y-axis values, since the $^{206}Pb/^{238}U$ ratio grows more slowly than the $^{207}Pb/^{235}U$. The disparity in half-lives can also be seen in the curling of the graph toward parallelism with the $^{207}Pb/^{235}U$ axis.



6. Discordia/Concordia

The U-series concordia/discordia technique allows us to determine not only the original crystallization age of a rock, but also the age of last metamorphism. In the diagram below, a rock moving along the concordia diagram from its initial time of formation 4.5 billion years ago lost lead at 3.5 billion years ago. Lead loss occurred along the red dashed line connected to the origin of the graph (a sample that lost all of its lead would plot at the origin itself). After the episodic lead loss, the rock evolved along a new concordia parallel to the initial one toward point P'. This point lies along a discordia (dashed green line) connecting the age of the rock to the time that has elapsed since the lead loss occurred (4.5 billion years - 3.5 billion years = 1.0 billion years).



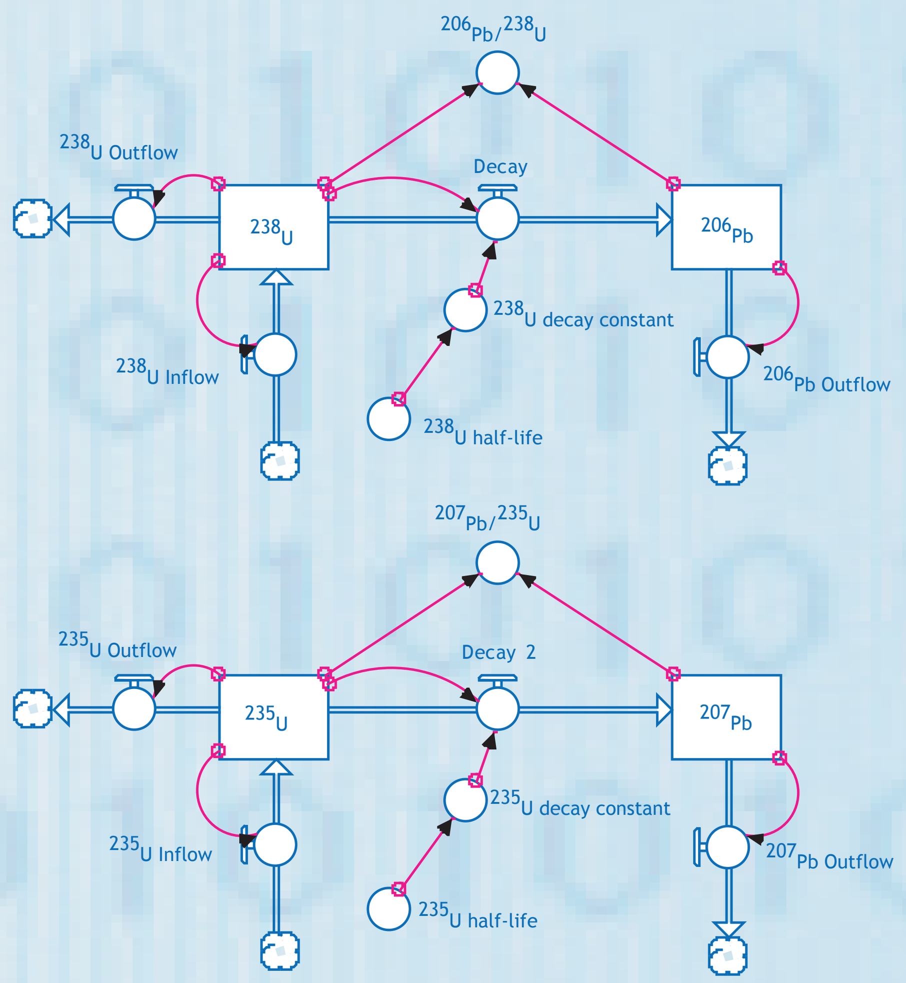
7. Modeling Discordia in STELLA

To simulate a metamorphic event that results in lead loss, uranium loss, or uranium gain, outflow and inflow arrows are added to the simple U-series concordia model. Outflows and inflows are activated using an if-then statement, and allow varying amounts of each isotope to be subtracted or added.

For example, the logical statement governing lead loss might read as follows:

IF(TIME=1500)THEN(0.1*Lead_206/DT)ELSE(0)

This statement removes 10% of the total ^{206}Pb reservoir at time = 1500 million years. Similar statements govern uranium loss or addition.

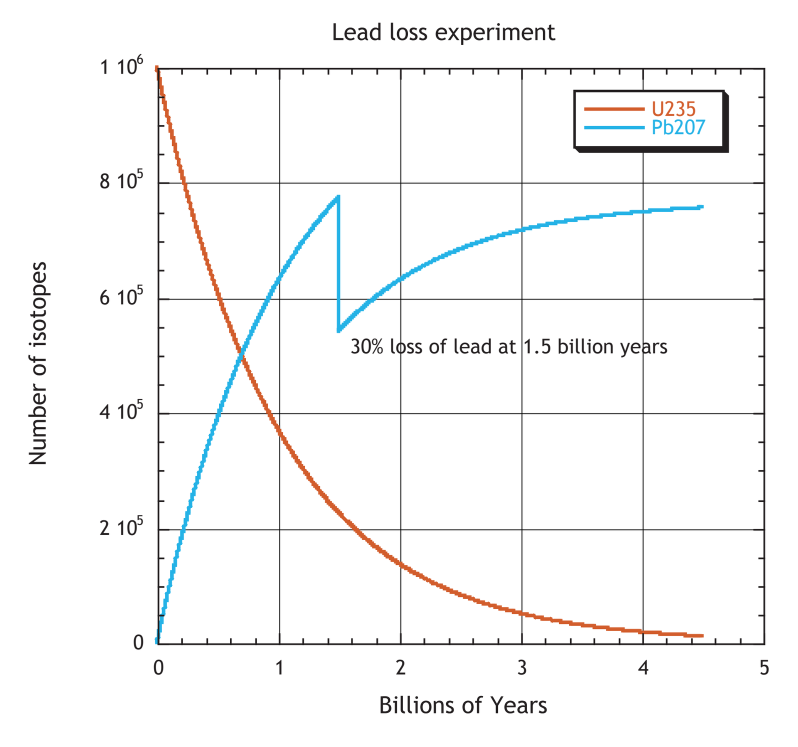


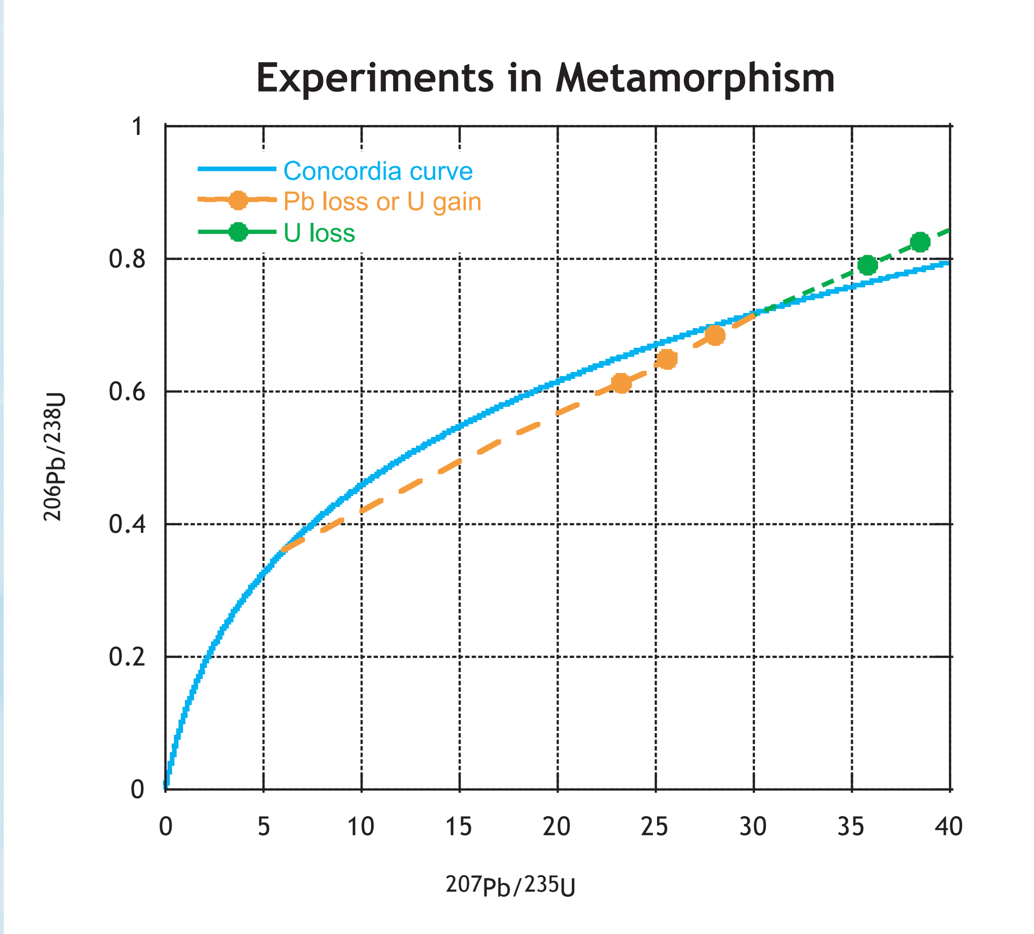
8. Results

The figure below shows the result of a model run in which 30% of the lead that had accumulated by 1.5 billion years was removed instantaneously. By the end of the 4.5 billion year run, the 207Pb/235U ratio is lower than if no lead loss had occurred.

The figure to the right shows the results of a series of experiments in which varying amounts of lead or uranium were lost or gained. Lead loss and uranium gain are effectively the same thing, and both create points that fall below the discordia curve. The experiments shown here were for 10%, 20%, and 30% lead loss at 1.5 billion years into a 3.5 billion year model run. The points define the discordia line, which projects to the time of original crystallization of the rock (in this case 3.5 billion years) and the time since metamorphism occurred (2 billion years ago).

Experiments with uranium loss lead to points that plot above the concordia curve, but when a straight line is extrapolated through these points, it intersects the concordia curve in the same locations, because the timing of original crystallization and metamorphism remains the same between the lead loss and uranium loss experiments.





References

- Dalrymple, G.B., 1991, The Age of the Earth, Stanford, CA, Stanford University Press, Chapter 3: Modern Radiometric Methods: How They Work.
- Faure, G., 1986, Principles of Isotope Geology (2nd Ed.), New York, John Wiley and Sons, Chapter 4: Radioactive Decay and Growth and Chapter 18: The U, Th-Pb Methods of Dating.
- http://www.ties.k12.mn.us/envision/student/lessons97/radioactivity

Acknowledgments:

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