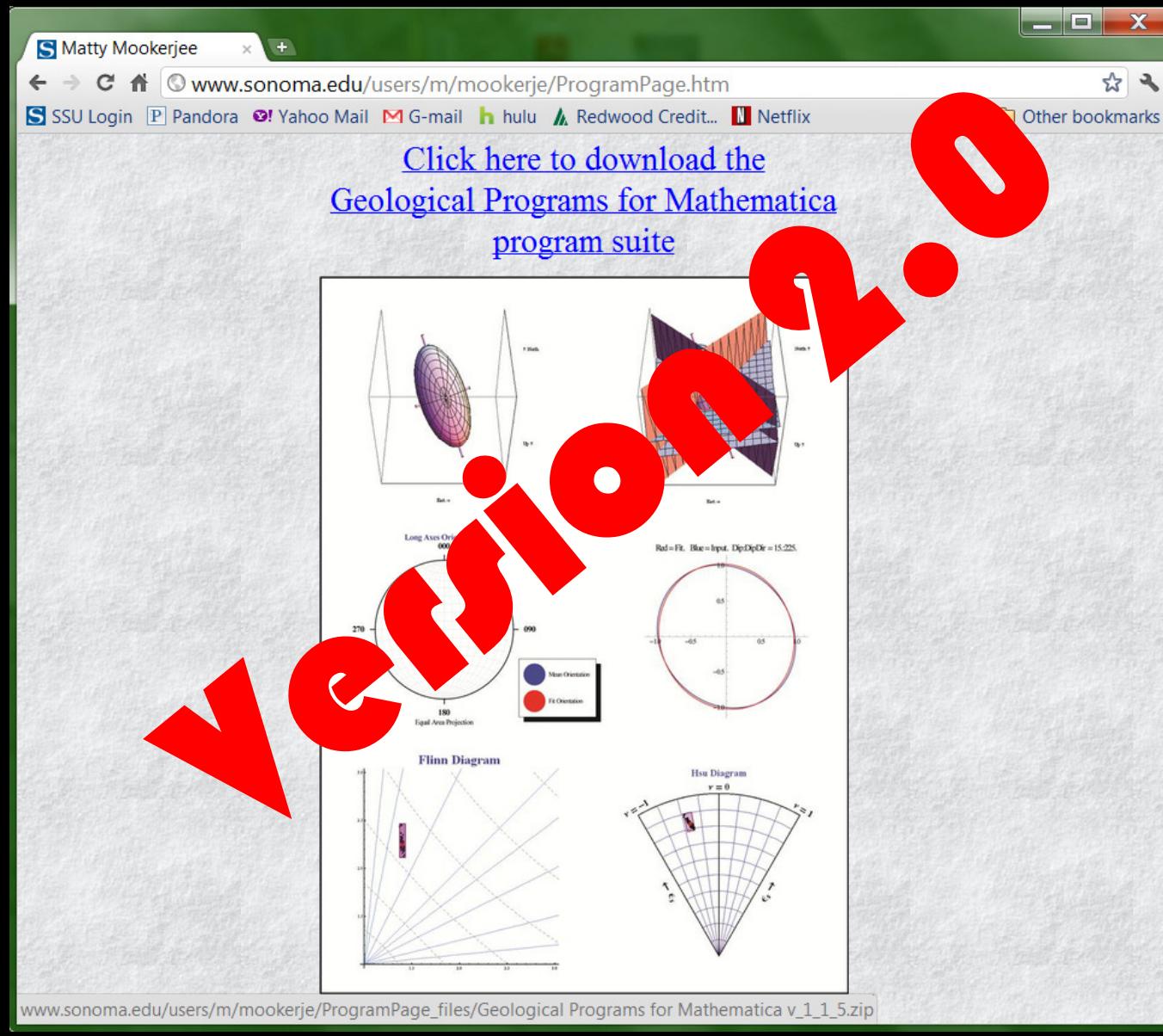


# Three-dimensional Strain Analysis using Mathematica

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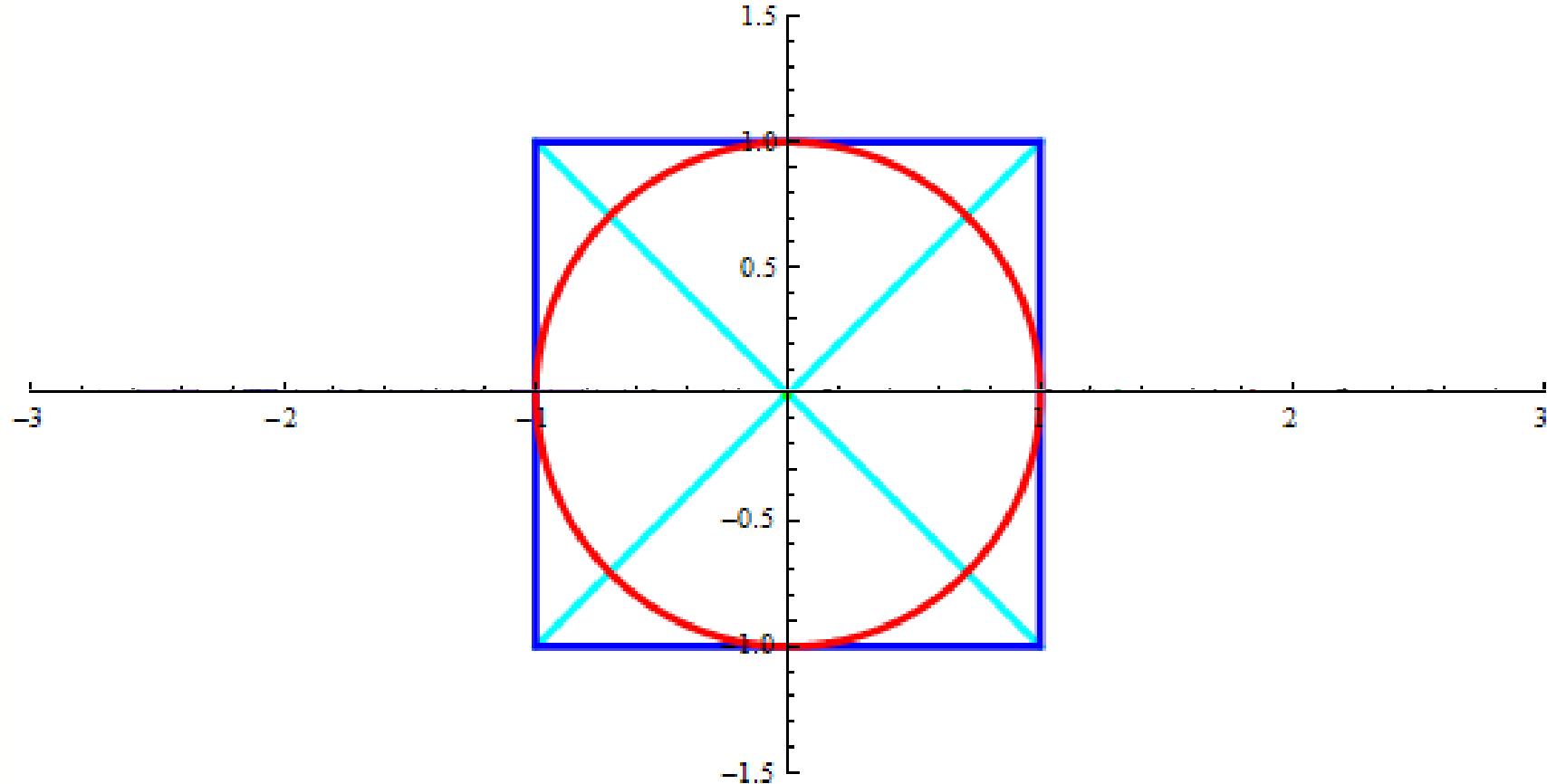
[www.sonoma.edu/users/m/mookerje/ProgramPage.htm](http://www.sonoma.edu/users/m/mookerje/ProgramPage.htm)



increment



Axial Ratio = 1. ;  $\phi = \text{Phi}[0.]$  ;  $\Psi = \psi[0.]$

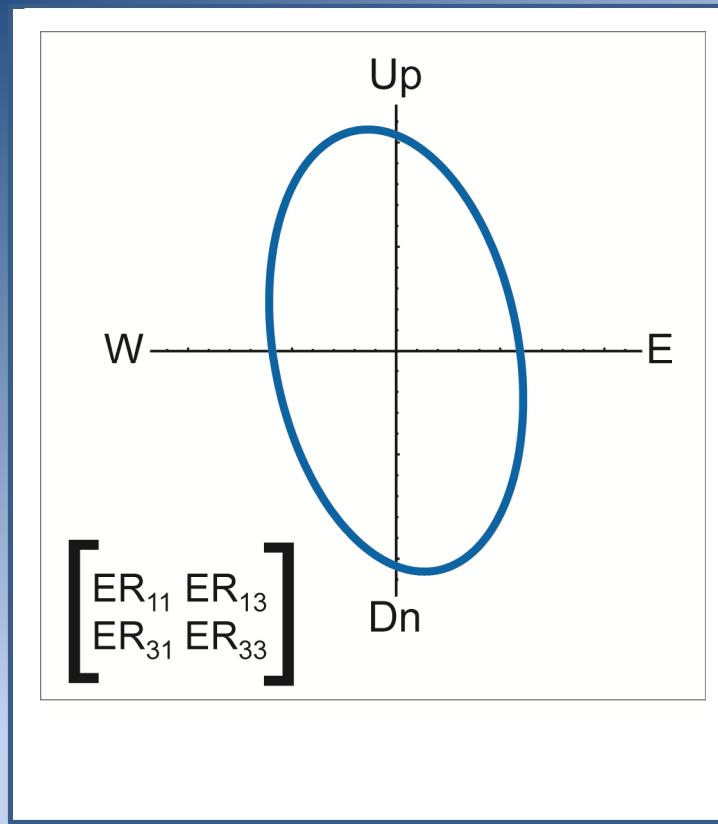


**Section Data th  
2D Pure vs Sim**

Plot Range for Lode's Ratio is -1 to 1

Lode's Ratio contour interval is 0.25

$$GE = \begin{bmatrix} \lambda_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \lambda_{yy} & \gamma_{yz} \\ \gamma_{xz} & \gamma_{yz} & \lambda_{zz} \end{bmatrix}$$



$$ER = \begin{bmatrix} \cos[DipDir - 90^\circ] & -\sin[DipDir - 90^\circ] & 0 \\ \sin[DipDir - 90^\circ] & \cos[DipDir - 90^\circ] & 0 \\ EM \sin[90 - Dip] & \cos[90 - Dip] & 1 \end{bmatrix} \cdot \left( \begin{bmatrix} ER \cdot \begin{bmatrix} \cos[DipDir - 90^\circ] & \sin[DipDir - 90^\circ] & 1 \\ \sin[DipDir - 90^\circ] & \cos[DipDir - 90^\circ] & 0 \\ ER^{11} & ER^{13} & ER^{31} & ER^{33} \end{bmatrix}^T & Det \begin{bmatrix} \cos[DipDir - 90^\circ] & \sin[DipDir - 90^\circ] & 1 \\ \sin[DipDir - 90^\circ] & \cos[DipDir - 90^\circ] & 0 \\ ER^{11} & ER^{13} & ER^{31} & ER^{33} \end{bmatrix} \end{bmatrix} \right)$$

(Mookerjee & Nickleach, 2011 – JSG)

# Assembling the Error Function

$$DM = \begin{bmatrix} \cos[\phi] & -\sin[\phi] \\ \sin[\phi] & \cos[\phi] \end{bmatrix} \cdot \begin{bmatrix} \sqrt{\frac{1}{Rf^2}} & 0 \\ 0 & \frac{1}{\sqrt{\frac{1}{Rf^2}}} \end{bmatrix} \cdot \begin{bmatrix} \cos[\phi] & \sin[\phi] \\ -\sin[\phi] & \cos[\phi] \end{bmatrix}$$

$$\sum_{i=1}^n (DM_{11,i} - EM_{11,i})^2 + (DM_{12,i} - EM_{12,i})^2 + (DM_{22,i} - EM_{22,i})^2$$

$n$  = number of sectional data planes

Minimize this function in terms of the elements  
of the general ellipsoid matrix,  $\lambda_{xx}$ ,  $\lambda_{yy}$ ,  $\lambda_{zz}$ ,  $\gamma_{xy}$ ,  $\gamma_{xz}$ ,  $\gamma_{yz}$

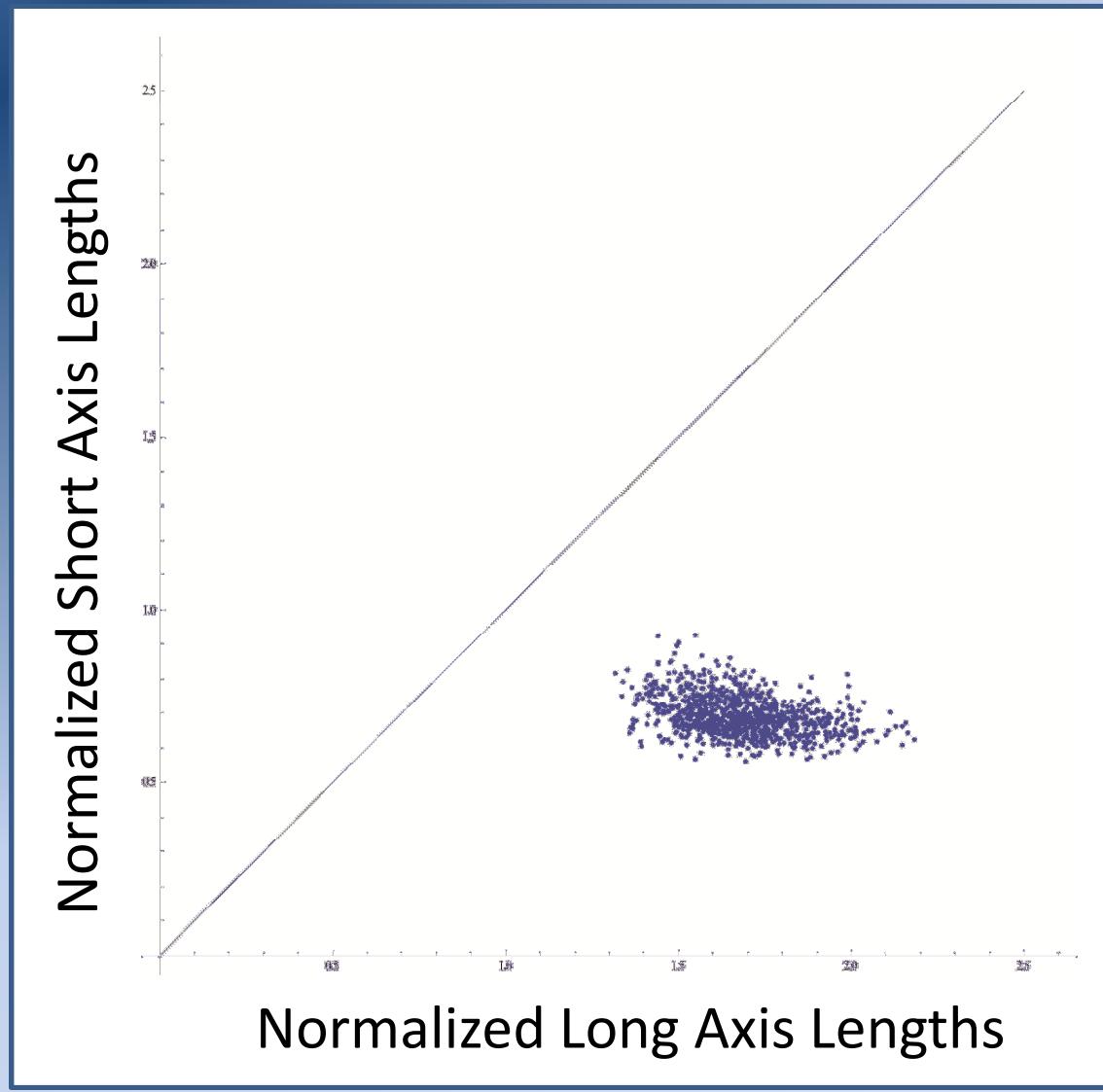
# Input Data Screen Shot

The screenshot shows a Microsoft Excel spreadsheet titled "Sample Strain Data for Stats.xls [Compatibility Mode] - Microsoft Excel". The data is organized into sections and rows, with some cells containing formulas or specific values.

	A	B	C	D	E	F	G	H	I	J
1	<b>Section #1</b>	of:	<b>8</b>							
2	<b>Dip</b>	<b>Dip Direction</b>	<b>Measurement Error</b>							
3	90	90	1							
4	<b>Long Axis</b>	<b>Short Axis</b>	<b>Angle</b>							
5	3.40402		1	-63.4675						
6	3.30707446		1	-65.84164655						
7	3.27975472		1	-62.27171413						
8	3.18622321		1	-61.62402772						
9	3.41664597		1	-63.03749337						
10	3.8270464		1	-64.13161541						
11	3.18803234		1	-65.51987395						
12	3.10714636		1	-62.47460386						
13	3.41780946		1	-62.77188823						
14	3.47907475		1	-64.49563413						
15	3.85077164		1	-63.81478991						
16	<b>Section #2</b>									
17	<b>Dip</b>	<b>Dip Direction</b>	<b>Measurement Error</b>							
18	90	135	1							
19	<b>Long Axis</b>	<b>Short Axis</b>	<b>Angle</b>							
20	3.3888		1	-45						
21	3.1081405		1	-44.32176045						
22	3.18082114		1	-45.94337513						
23	3.69744317		1	-48.06718461						

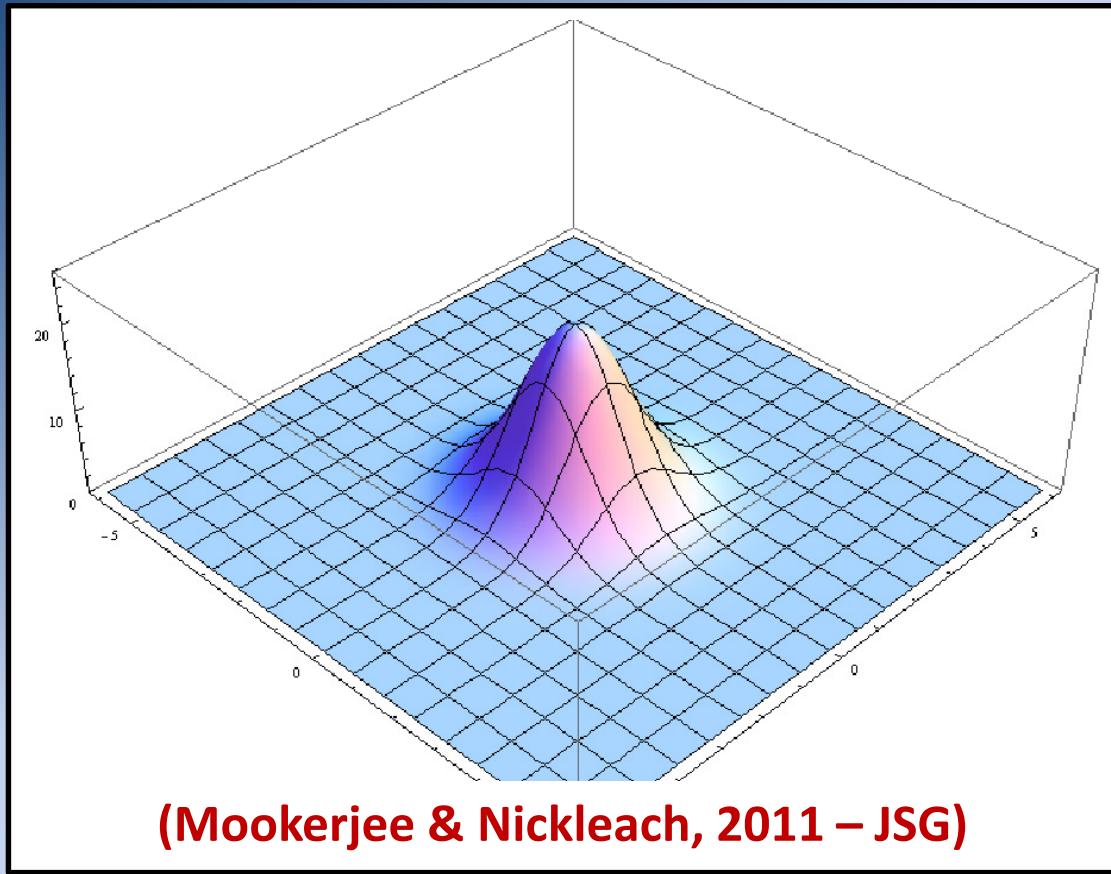
# Statistical Analysis

## Kernel Density Estimation



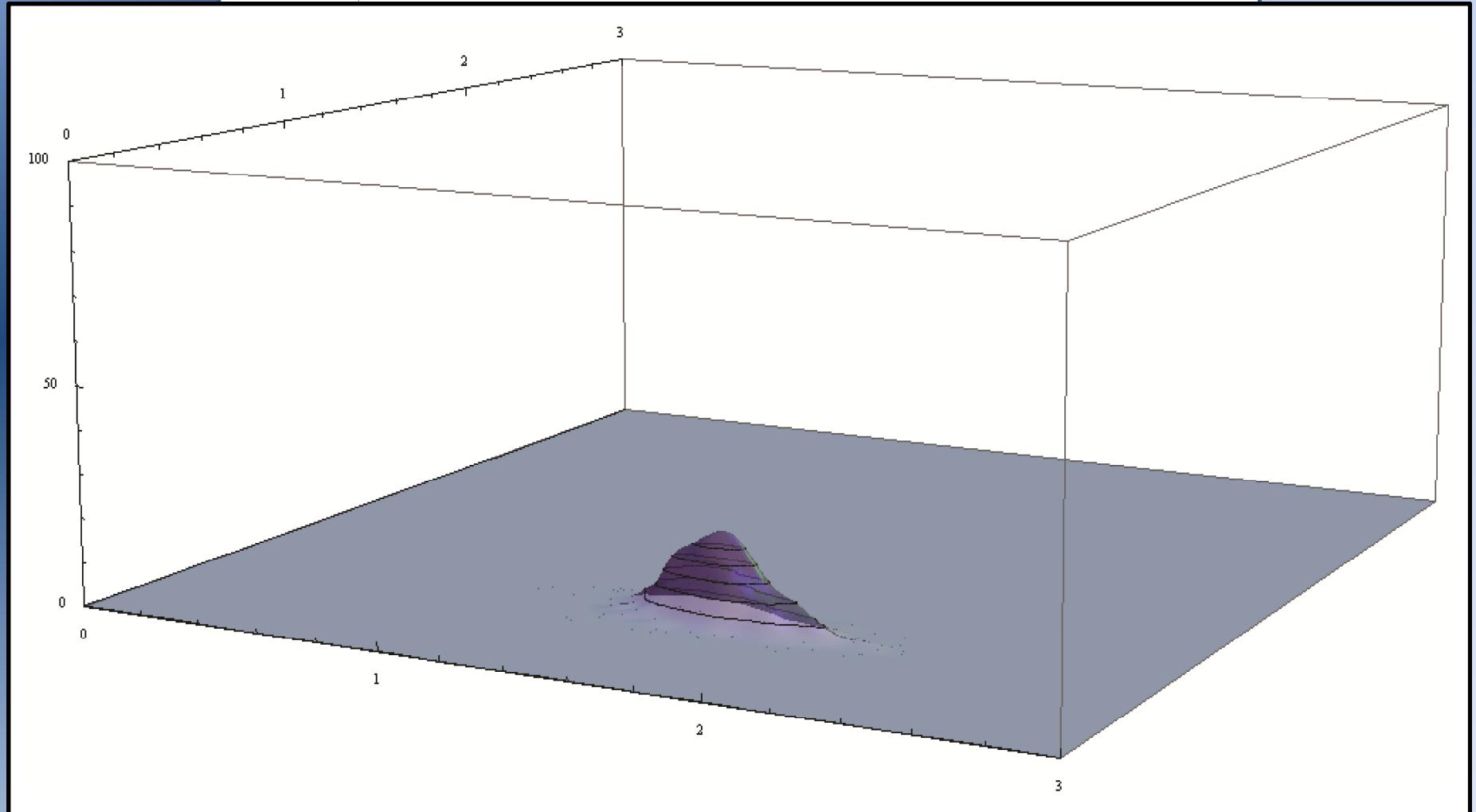
# Kernel Density Estimation

$$KDE[\lambda_1, \lambda_3] = \frac{1}{2\pi \times N \times h_1 \times h_3} * \sum_{i=1}^N \exp \left[ -\frac{1}{2} \left( \left( \frac{\lambda_1 - L_{1,i}}{h_1} \right)^2 + \left( \frac{\lambda_3 - L_{3,i}}{h_3} \right)^2 \right) \right]$$



$$h_i = 4 \times 1.06 \times \min\{\sqrt{Var(L_i)}, IQR_i / 1.34\} \times N^{-1/5}$$

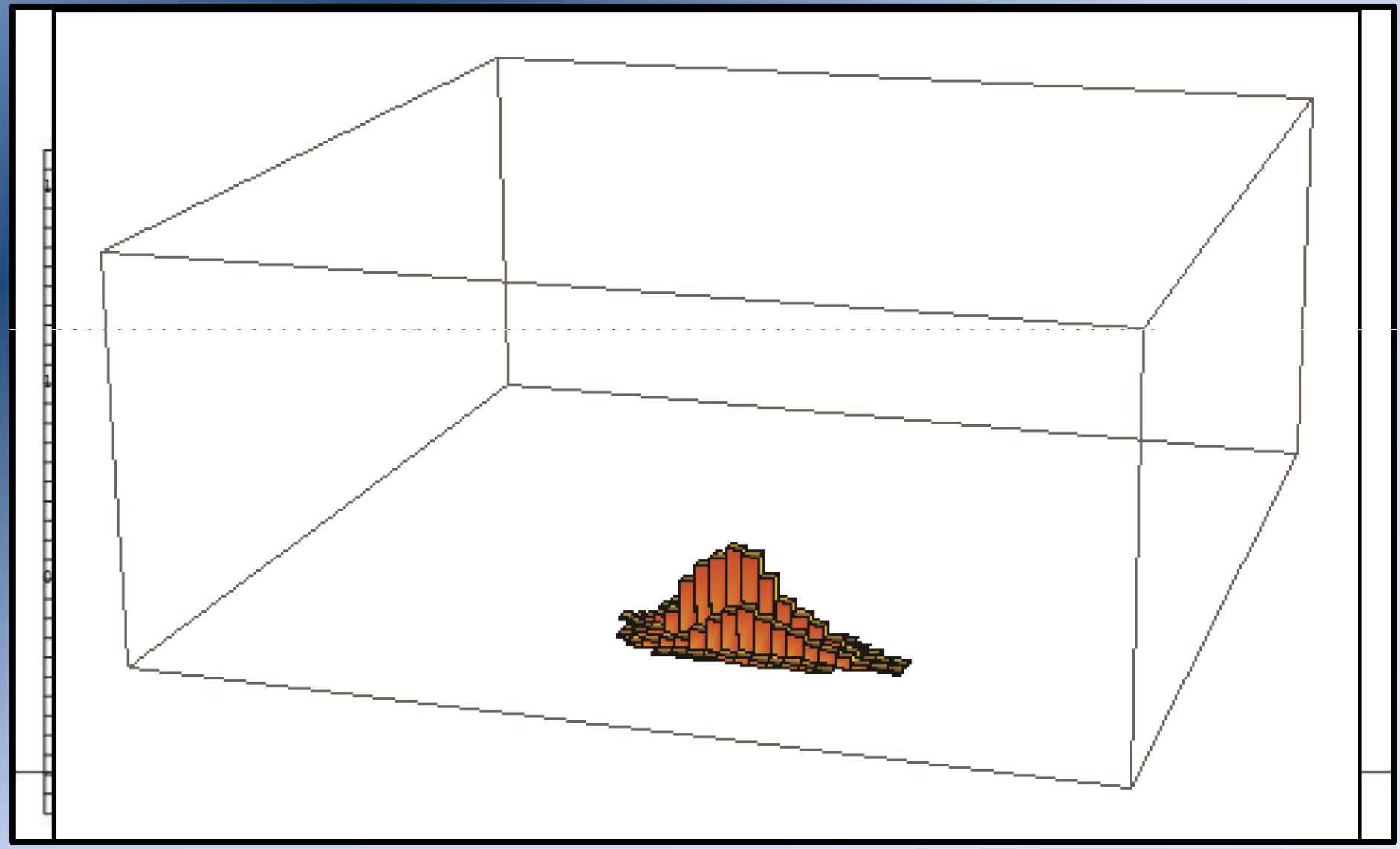
# Kernel Density Estimation



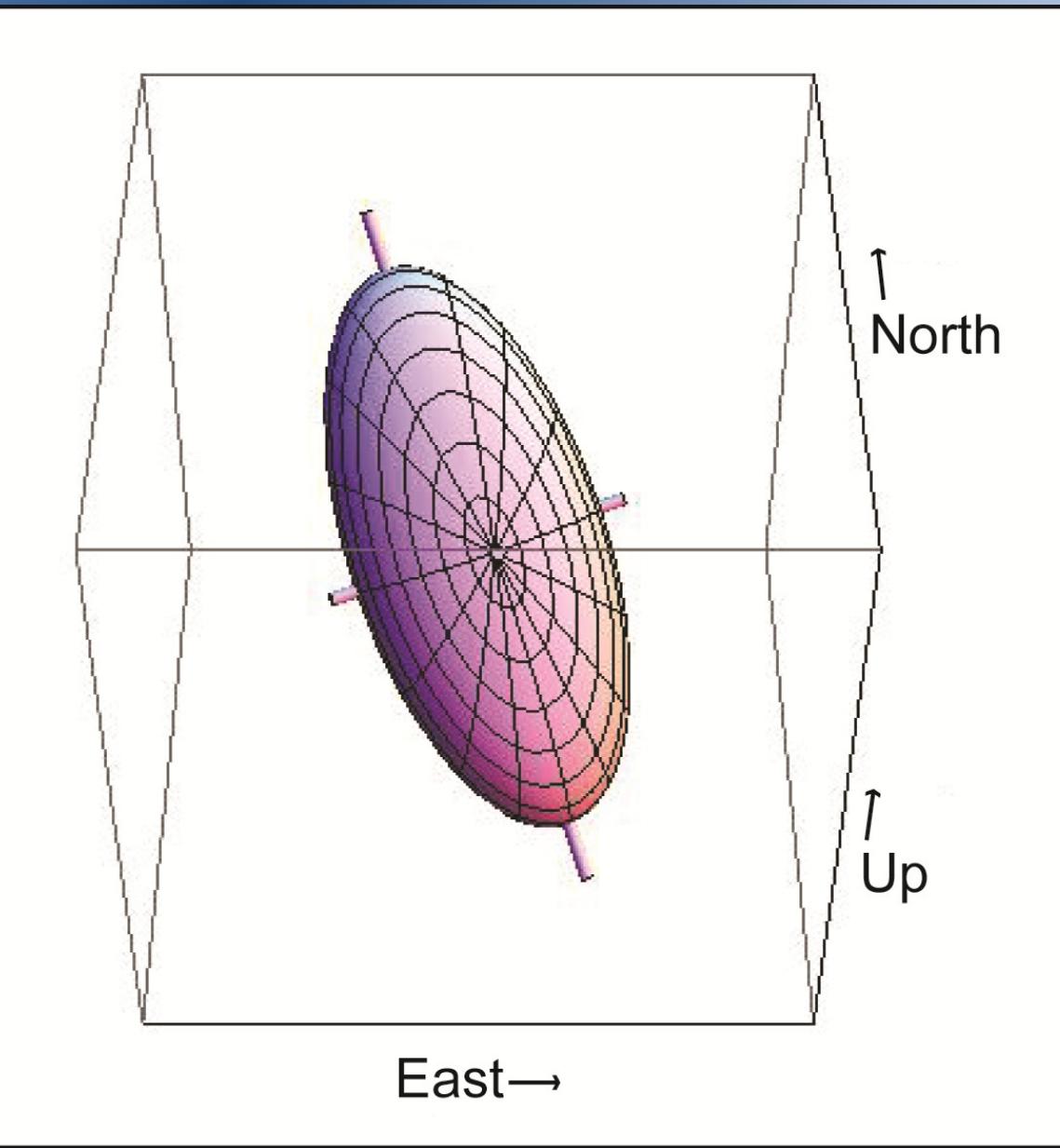
Normalized Long Axis Lengths

# Kernel Density Estimation

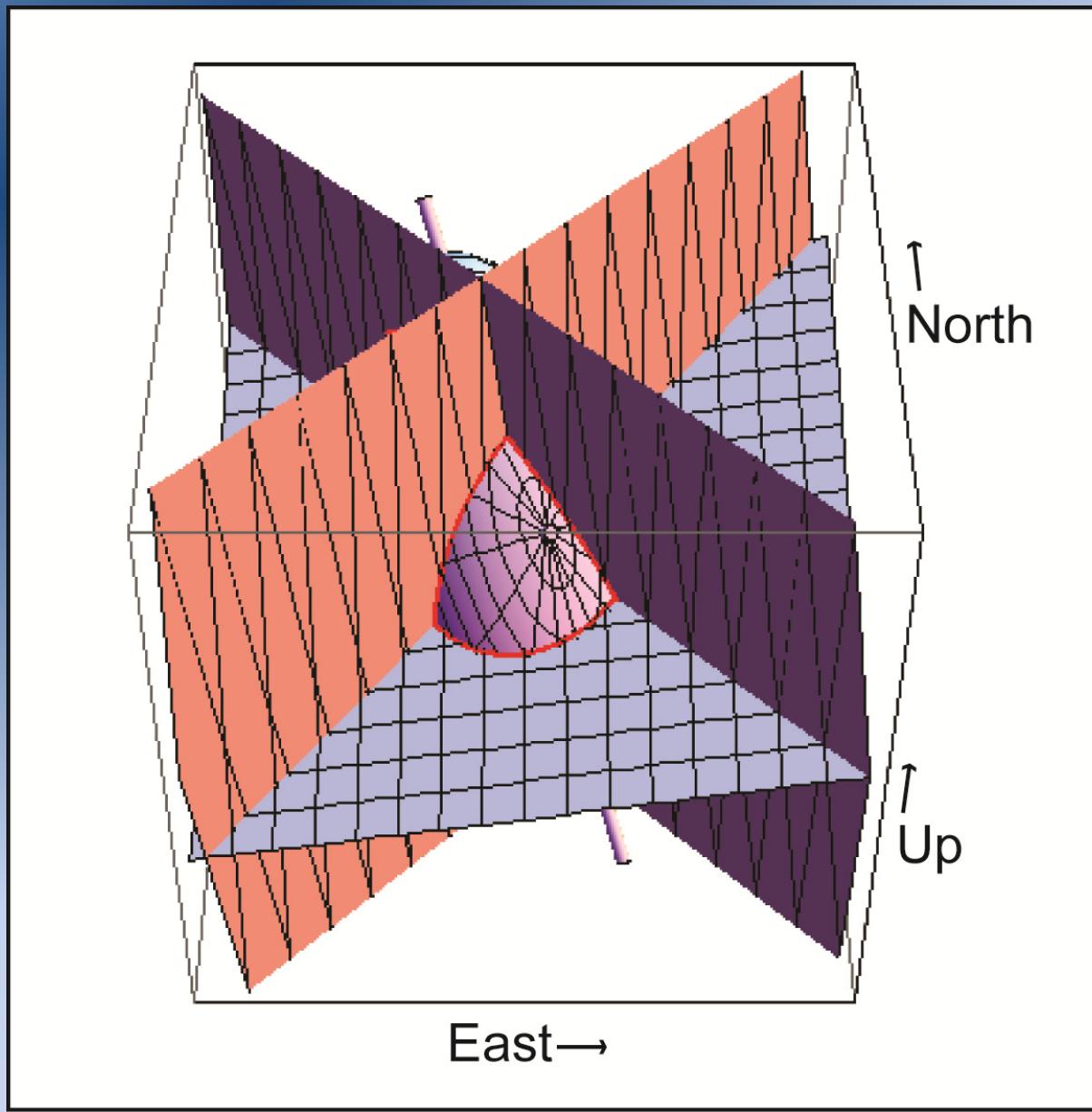
## 95% Volume



# Graphical Output

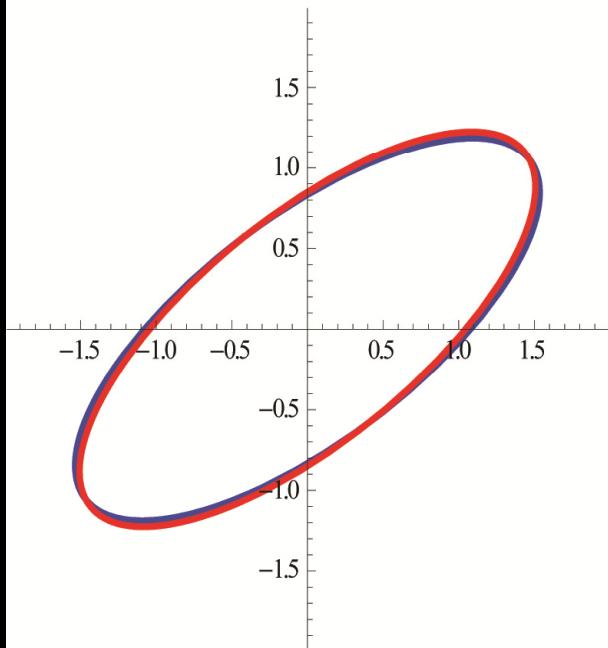


# Graphical Output

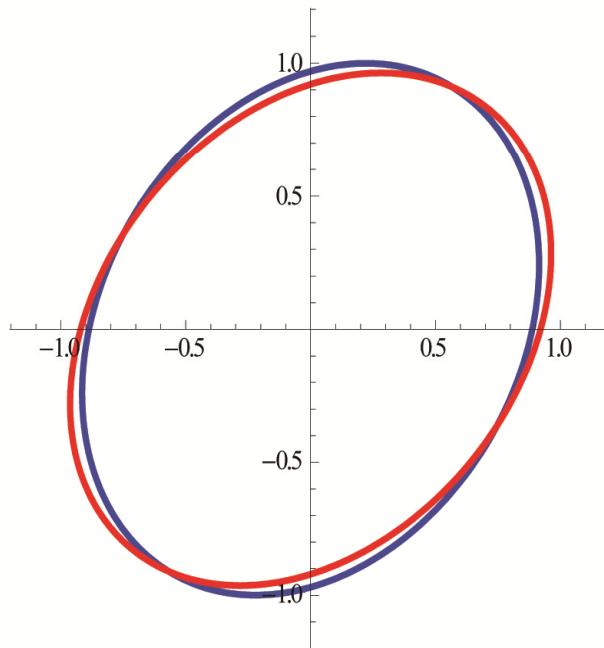


# Graphical Output

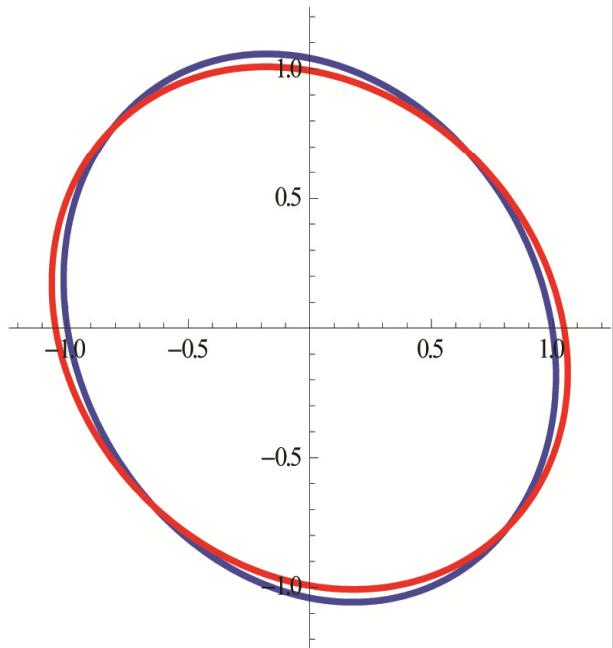
Red = fit. Blue = input. Dip:DipDir = 85.:45.



Red = fit. Blue = input. Dip:DipDir = 85.:135.

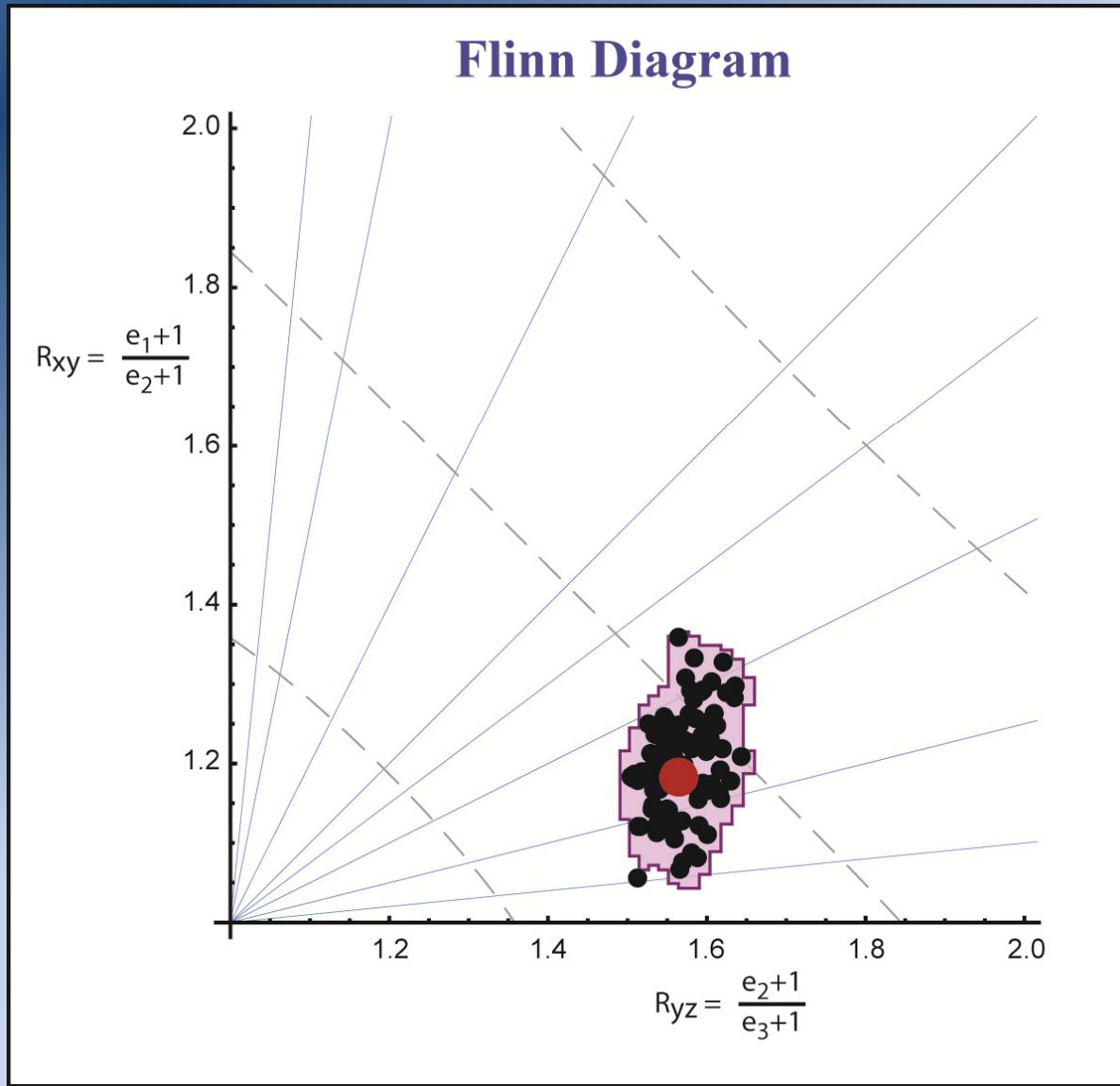


Red = fit. Blue = input. Dip:DipDir = 15.:225.



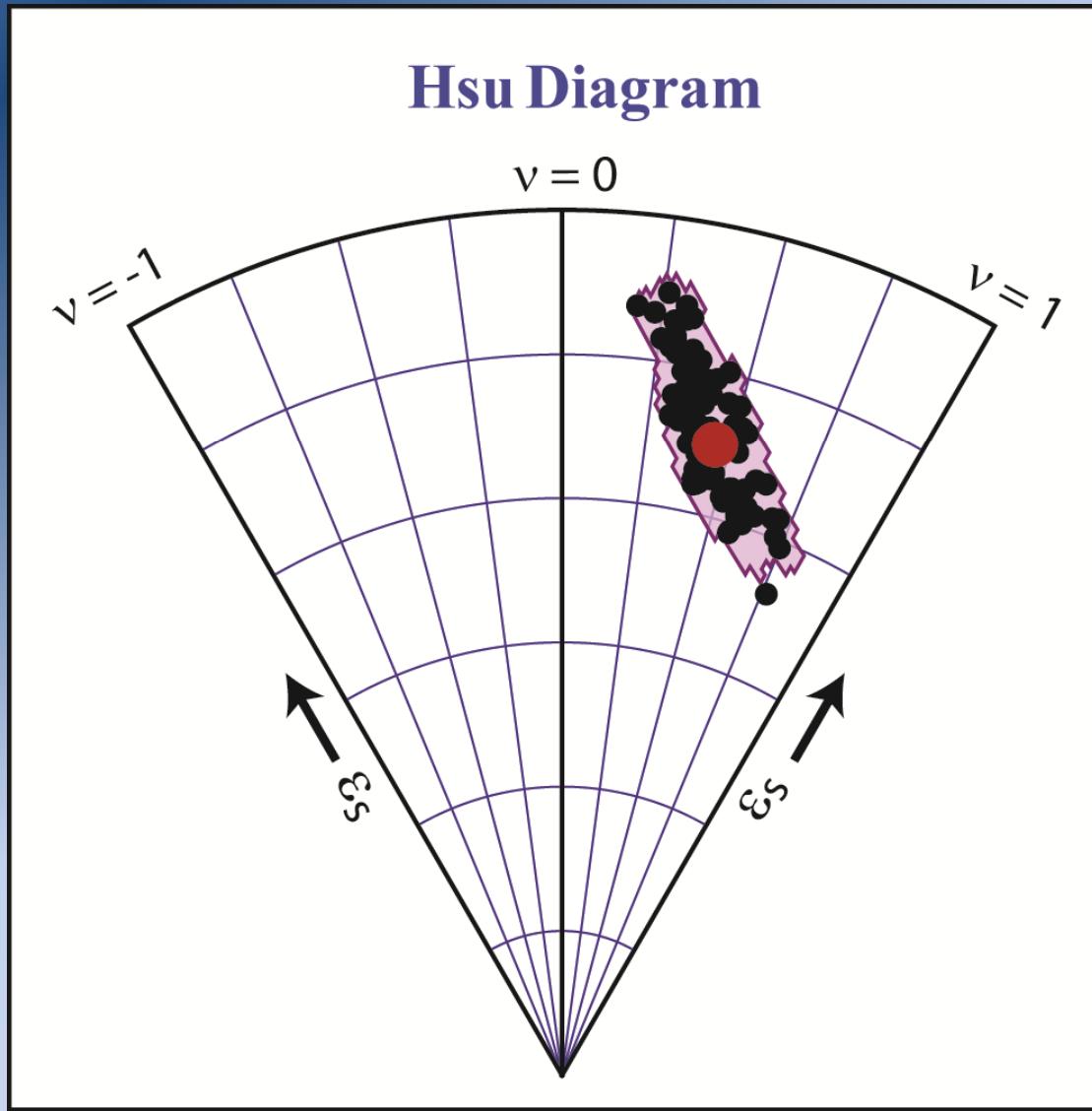
# Graphical Output

## Flinn Diagram with Error Region



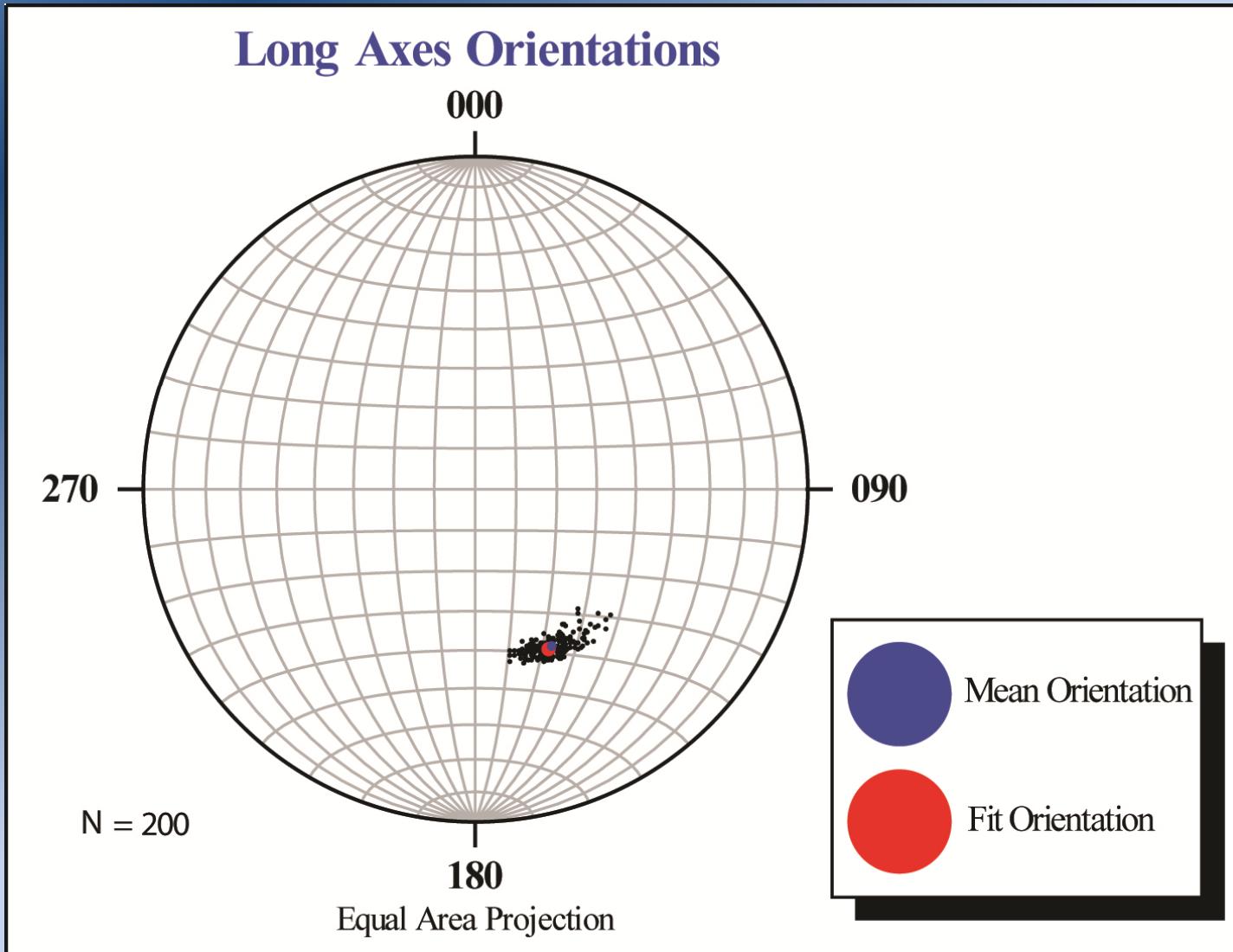
# Graphical Output

## Hsu Diagram with Error Region

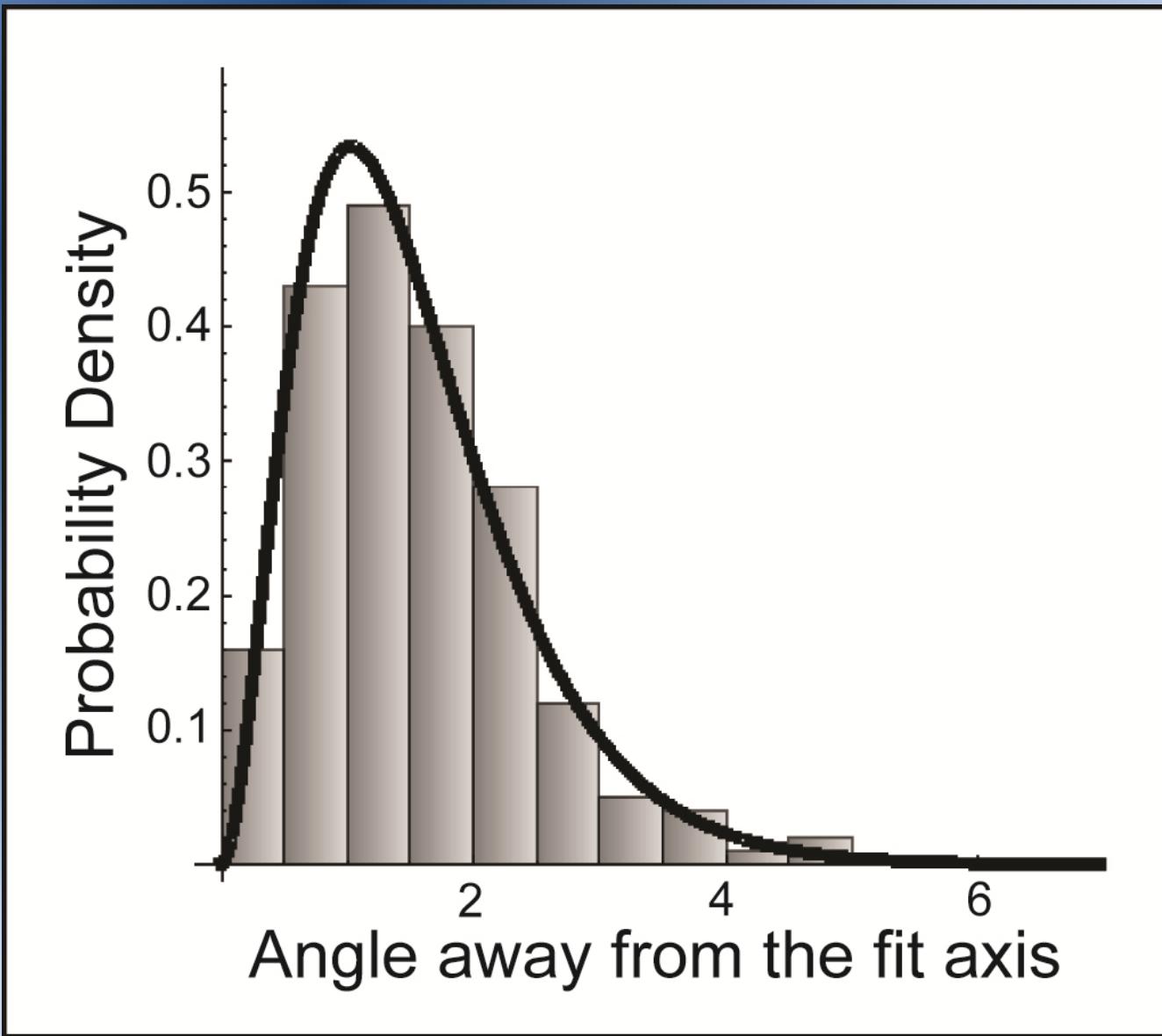


# Graphical Output

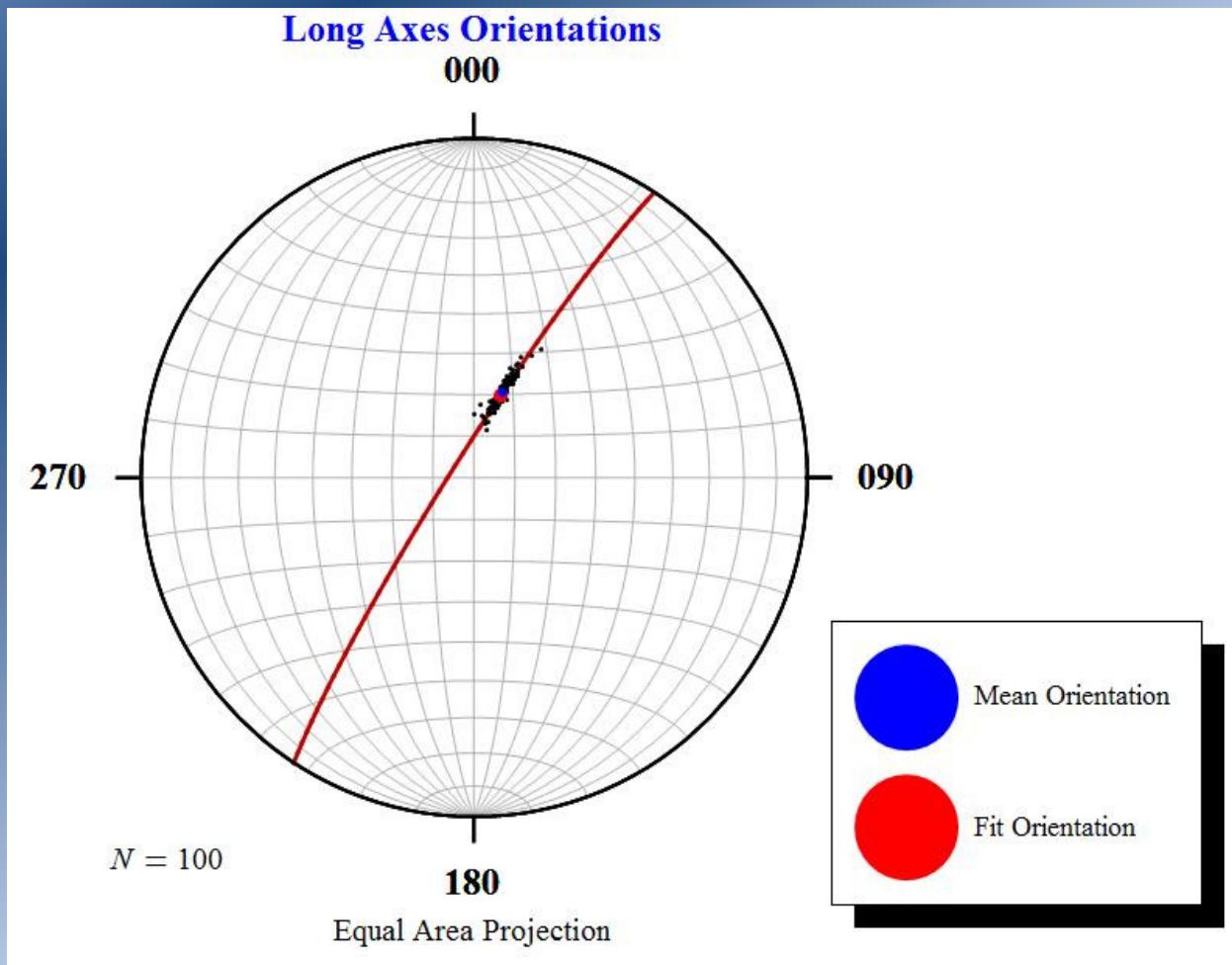
## Axes Orientation Data



# Best-Fit Gamma Distributions



# Best-Fit Plane for Axes Distributions



The Trend:Plunge of the Mean Orientation =  $18.9049^\circ:68.325^\circ$

The Trend:Plunge of the Fit Orientation =  $18.2759^\circ:69.1741^\circ$  with a 95% error angle =  $9.22132^\circ$

The Dip:Dip Direction of the Fit Plane =  $84.5979^\circ:302.759^\circ$  and a Mean Difference Angle of:  $0.657062^\circ$

# Example Data Set with Error Regions

