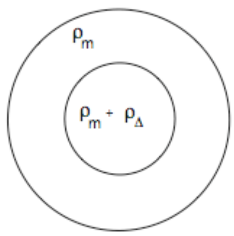
Donald Forsyth, Dept. of Geological Sciences, Brown University, Providence, RI 02912
Donald_Forsyth@brown.edu
Geo 161
Use mass and moment of inertia of the earth to constrain the density distribution of the earth.

a. Assume the earth is radially symmetric and composed of 2 layers - a uniform outer mantle and a uniform core. Neglect the earth's crust. Rocks we think are representative of the earth's mantle have a density of about 3300 kg/m^3 at the earth's surface. Assuming that this is the density of the mantle, ρ_m, compute the radius and density of the core. You will need the mass of the earth, M_e = 5.976 x 10^{24} kg and moment of inertia, $I_e = .3308\ M_e\ a^2$. Radius of earth, a = 6.37 x 10^6 m. Simplest approach is to assume a uniform sphere with density ρ_m and the radius of the earth, then consider radius of inner uniform sphere (core) with additional density, ρ_Δ, such that the density of the core $\rho_c = \rho_m + \rho_\Delta$. The sum of the effects of the two uniform spheres have to satisfy two constraints, the total mass and total moment of inertia, thus giving two equations in the two unknowns, r_{core} and ρ_Δ.



b. 3300 kg/m^3 is probably the minimum density of the mantle, since rocks are compressed with depth by increasing pressure. The result of part (a) then gives the maximum possible radius of the core. What is the maximum possible density of the outer shell consistent with the mass and moment of inertia constraints? (let $r_{core} \rightarrow 0$ and $\rho_\Delta \rightarrow \infty$). You will have to think here about how to satisfy both constraints simultaneously

$$M_e = \frac{4}{3}\pi\rho_m a^3 + \frac{4}{3}\pi\rho_\Delta r_{core}^3$$
$$I_e = \frac{8}{15}\pi\rho_m a^5 + \frac{8}{15}\pi\rho_\Delta r_{core}^5$$

c. From seismology, we know that the radius of the core is about 3500 km -- what are the densities of the core and mantle in this case?

d. What are the mass and moment of inertia in a sphere in which density increases linearly with increasing depth (i.e., $\rho(r) = c - br$)? (Evaluating integrals analytically is all that is required here, no numbers).

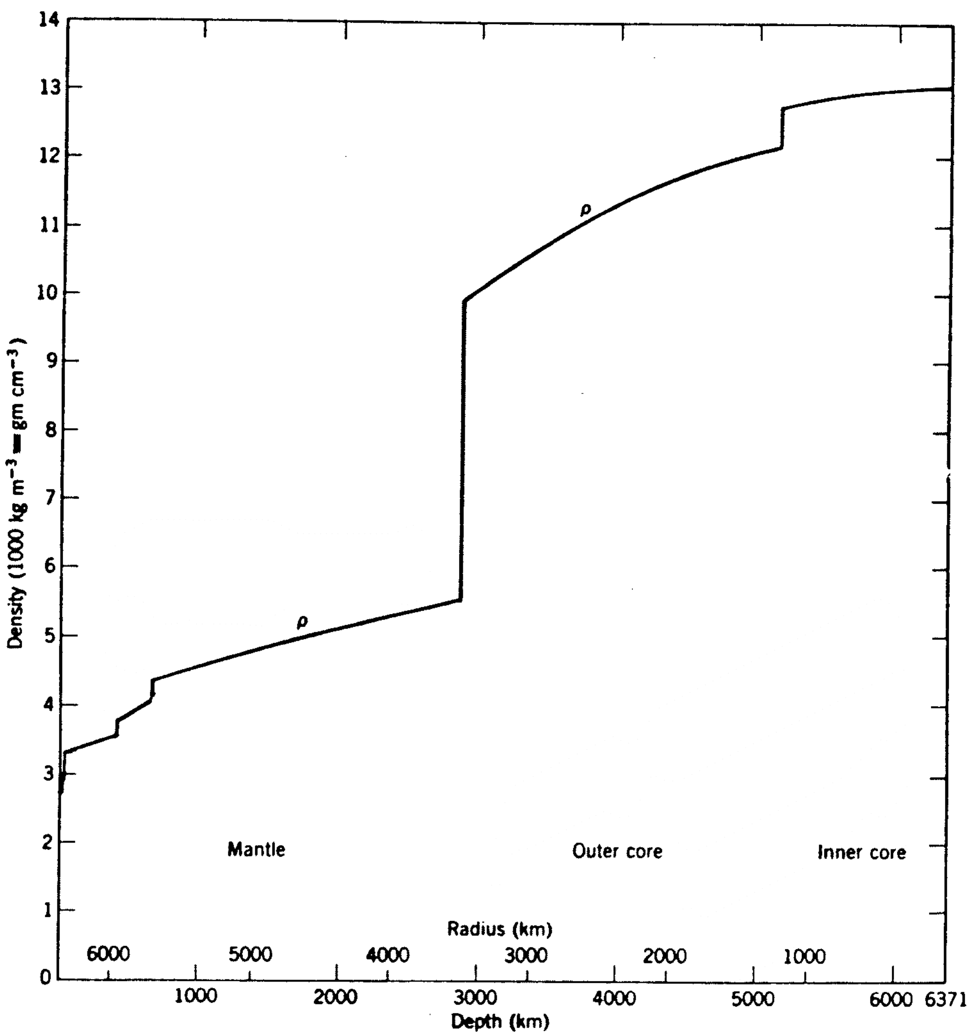
e. Assume the radius of the core = 3500 km. Assume the density in both the core and the mantle increase linearly with depth with the same gradient, and that the density at the surface (r = a) is 3300 kg/m^3. What is the profile of density in the core and the mantle? Use your analytical expression from part d for whole earth and add the extra effect of uniform extra core density, as in part a.

This part requires the solution of three equations in three unknowns, c, b_m, and ρ_Δ. Set up in form

$$c - b_m a = 3300$$
$$M_m + M_c = M_e$$
$$I_m + I_c = I_e$$

where M_m and I_m, the mass and moment of inertia of the whole earth not counting the extra effect of increased density in the core, are functions of b_m and c, and M_c and I_c, the mass and moment of inertia of the <u>extra</u> density of the core, are functions of ρ_Δ.

f. How do your models a, c, and e compare to detailed density profiles of the earth? (i.e., plot on diagram below and discuss briefly.)



Density profile of Earth model by Dziewonski et al. (1975) (solid line)