Daisyworld Exercise – Instructor Notes

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The Daisyworld model created by Andrew Watson and James Lovelock (1983, Tellus, v. 35B, p. 284-289) is a wonderful example of a self-regulating system incorporating positive and negative feedbacks. The model consists of a planet on which black and white daisies are growing. The growth of these daisies is governed by a parabolic shaped growth function regulated by planetary temperature and is set to zero for temperatures less than 5 ºC or greater than 40 ºC and optimized at 22.5 ºC. The model explores the effect of a steadily increasing solar luminosity on the growth of daisies and the resulting planetary temperature. The growth function for the daisies allows them to modulate the planet's temperature for many years, warming it early on as black daisies grow, and cooling it later as white daisies grow. Eventually, the solar luminosity increases beyond the daisies' capability to modulate the temperature and they die out, leading to a rapid rise in the planetary temperature. Several people have created STELLA models of Daisyworld, including Dave Bice at Carleton College and Dawn Wright at Oregon State University. I owe them thanks in coming up with this exercise as their models formed its foundation.

While the mathematics governing Daisyworld are fairly straightforward, I will go through the steps that Watson and Lovelock leave out of their paper for the benefit of anyone interested in going through the math with students.

The growth of daisies is governed by the expressions:

$$da_w/dt=a_w(x\beta_w-\gamma) \qquad \text{(Eqns. 1)}$$
$$da_b/dt=a_b(x\beta_b-\gamma)$$

where a_w and a_b are the areas of ground covered by white and black daisies, x is the proportion of bare ground still available for colonization, β is a growth function dependent on temperature, and γ is a death rate.

$$x=p-a_b-a_w \qquad \text{(Eqn. 2)}$$

where p is set equal to 1. X and the areas of ground covered by the daisies are therefore fractional areas less than 1. The growth function:

$$\beta_y = 1 - 0.003265*(22.5-T_y)^2 \qquad \text{(Eqn. 3)}$$

where T_y is the temperature of the black or white daisies. Since the white and black daisies have different albedos, they have different temperatures, and for this reason, each color must have its own β_y growth function. The value for β_y falls below zero for temperatures lower than 5 degrees and higher than 40 degrees and so has to be set to equal zero for these cases when creating the STELLA model (see the student exercise and answer key).

Because of its parabolic shape, the growth function acts as both a positive and a negative feedback. Initially, as the planet warms due to the increasing luminosity of the sun, a small population of black daisies warms sufficiently to begin growing. As these daisies grow, their presence further warms the planet due to their low albedo, leading to still more growth in a positive feedback loop. Due to the parabolic shape of the growth function, however, continued growth of new black daisies slows as temperatures become too warm for them. Eventually, temperatures are warm enough to allow the white daisies to begin to grow despite their high albedos. Their growth acts as a negative feedback. The more these daisies grow, the more they act to cool the warming planet.

The energy balance of Daisyworld can be determined by equating the amount of energy coming in from the sun with the amount of energy going out via radiation:

$$SL*(1-A) = \sigma(T_d+273)^4 \qquad \text{(Eqn. 4)}$$

where S is the solar constant (917 W/m^2), L is a function that ramps up the solar output over time (0.5+0.02*time), A is the planetary albedo, σ is the Stefan-Boltzmann constant (5.67e-8 W/m^2K^4), and T_d is the temperature of Daisyworld.

The planetary albedo is derived by taking the weighted average of the albedos of the black and white daisies and of bare ground (A_w, A_b, and A_g are albedos; a_w, a_b, and x are fractional areas covered by daisies or bare ground):

$$A = A_w*a_w + A_b*a_b + A_g* x \qquad \text{(Eqn. 5)}$$

Watson and Lovelock determine the local temperature (T_y) of bare ground, white or black daisies from the expression:

$$(T_y +273)^4=q*(A-A_y)+(T_d+273)^4 \qquad \text{(Eqn. 6)}$$

where q is a positive constant I will describe below. To show that using this equation for local temperature is valid in terms of the overall energy balance for the planet, they perform the following calculations:

The total energy radiated by Daisyworld to outerspace (F) is equivalent to the sum of the energies radiated from each type of ground cover (i.e., bare ground, black daisies, or white daisies):

$$F = \sum_y a_y \sigma(T_y + 273)^4 \qquad \text{(Eqn. 7)}$$

substituting Equation 6 above into Eqn. 7 gives:

$$F = \sum_y a_y \sigma[q * (A - A_y) + (T_d + 273)^4]$$

so

$$F = \sum_y a_y \sigma q A - \sum_y a_y \sigma q A_y - \sum_y a_y \sigma (T_d + 273)^4$$

In this equation, σ, q, A, and T_d are all constants that can be pulled out of the summations to give:

$$F = \sigma q A \sum_y a_y - \sigma q \sum_y a_y A_y - \sigma (T_d + 273)^4 \sum_y a_y$$

but

$$\sum_y a_y = 1$$

because the sum of all of the areas on the planet is 1, and

$$\sum_y a_y A_y = A$$

because the planetary albedo, A, is equal to the area-weighted average albedo of the black and white daisies and of bare ground, so

$$F = \sigma q A - \sigma q A - \sigma (T_d + 273)^4$$

or

$$F = \sigma (T_d + 273)^4$$

Since the final value for F is equivalent to SL*(1-A), the planet can be seen to be in radiative balance using equation 6 for local temperature.

To understand the meaning of the q constant, Watson and Lovelock eliminate the planetary albedo term between Eqns. 4 and 6:

$$\sigma (T_d + 273)^4 = \mathrm{SL} * (1 - \mathrm{A}) \qquad \text{(Eqn. 4)}$$

so

$$A = 1 - \frac{\sigma (T_d + 273)^4}{SL}$$

but

$$(T_y + 273)^4 = q * (A - A_y) + (T_d + 273)^4 \qquad \text{(Eqn. 6)}$$

so

$$A = \frac{(T_y + 273)^4 - (T_d + 273)^4}{q} + \mathrm{A}_y$$

so setting these 2 equations equal

$$1 - \frac{\sigma (T_d + 273)^4}{SL} = \frac{(T_y + 273)^4 - (T_d + 273)^4}{q} + \mathrm{A}_y$$

multiplying both sides by q gives:

$$q * [1 - \frac{\sigma (T_d + 273)^4}{SL}] = (T_y + 273)^4 - (T_d + 273)^4 + A_y q$$

$$q * [1 - \frac{\sigma (T_d + 273)^4}{SL}] - A_y q + (T_d + 273)^4 = (T_y + 273)^4$$

$$(T_y + 273)^4 = q * (1 - A_y) + (T_d + 273)^4 * (1 - \frac{q\sigma}{SL}) \qquad \text{Eqn. 8}$$

In the case were q = 0, the planet is a perfect conductor of absorbed radiation. In other words, excess energy absorbed in areas covered by black daisies is conducted away to areas covered by white daisies to create a planet with a perfectly uniform distribution of surface energy.

Plugging q=0 into Eqn. 8 above gives

$(T_y+273)^4 = (T_d+273)^4$

In other words, the local temperature (T_y) is everywhere equivalent to the average planetary temperature (T_d).

If on the other hand q=SL/σ, then there is no heat transfer between the different daisy species or the ground. Plugging this into Eqn. 8 gives:

$$(T_y + 273)^4 = \frac{SL}{\sigma}(1 - A_y) + (T_d + 273)^4 * (1 - \frac{SL\sigma}{\sigma SL})$$

the latter half of this equation goes to 0, to give:

$$(T_y + 273)^4 = \frac{SL}{\sigma}(1 - A_y)$$

So in this situation, the local temperatures are determined by the local radiation balance.

Watson and Lovelock use a value intermediate between 0 and SL/σ for q to allow for some conduction from black daisy areas to white daisy areas. That value is 0.2*SL/σ.

Students experiment with changing the q value and with changing the albedos of white and black daisies and describe and explain their results.

The Daisyworld modeling exercise has several purposes:
1) To explore a self-regulating system
2) To learn how positive and negative feedbacks work
3) To convey the importance of “subroutines” in decreasing clutter in model structures